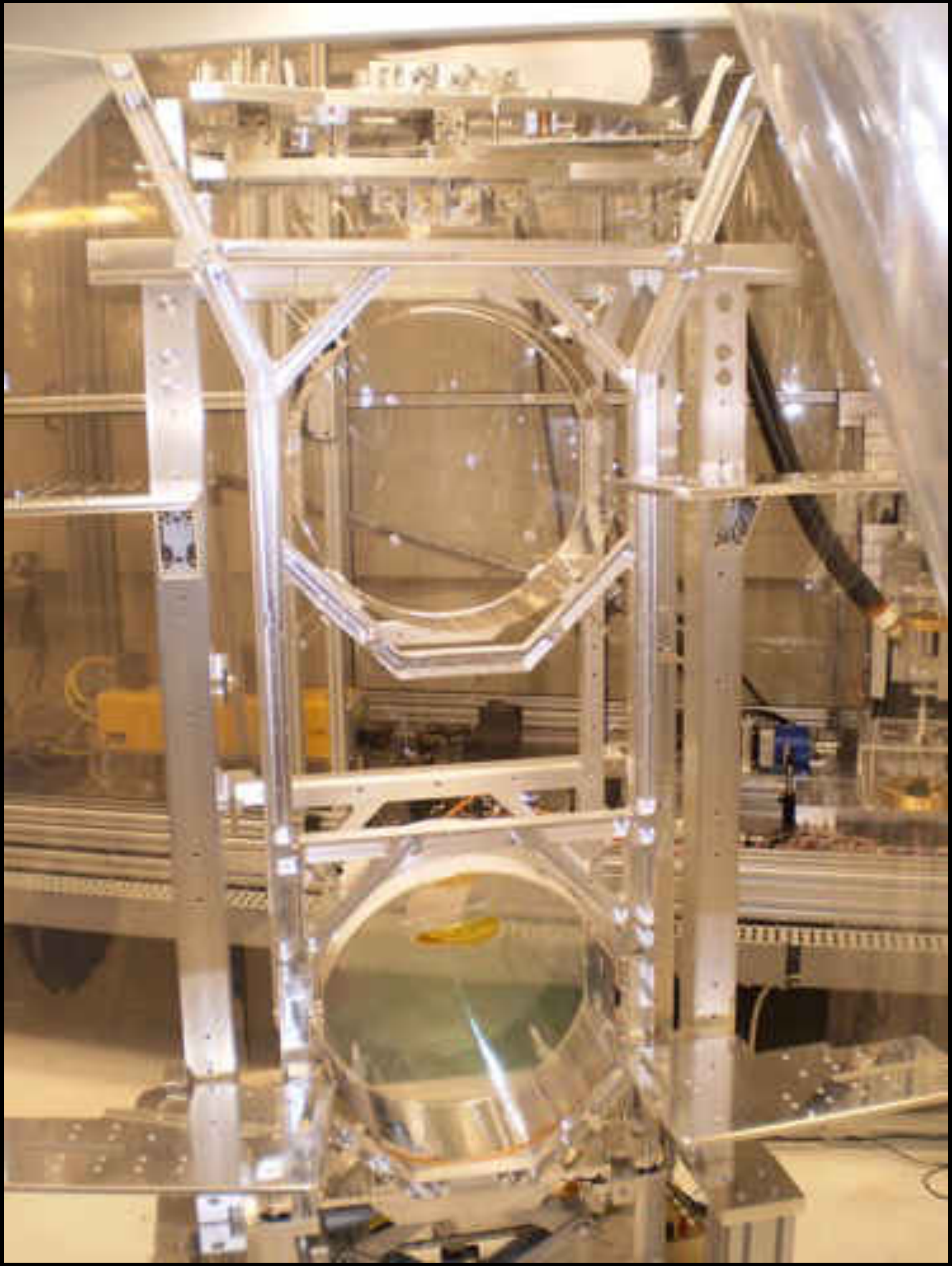
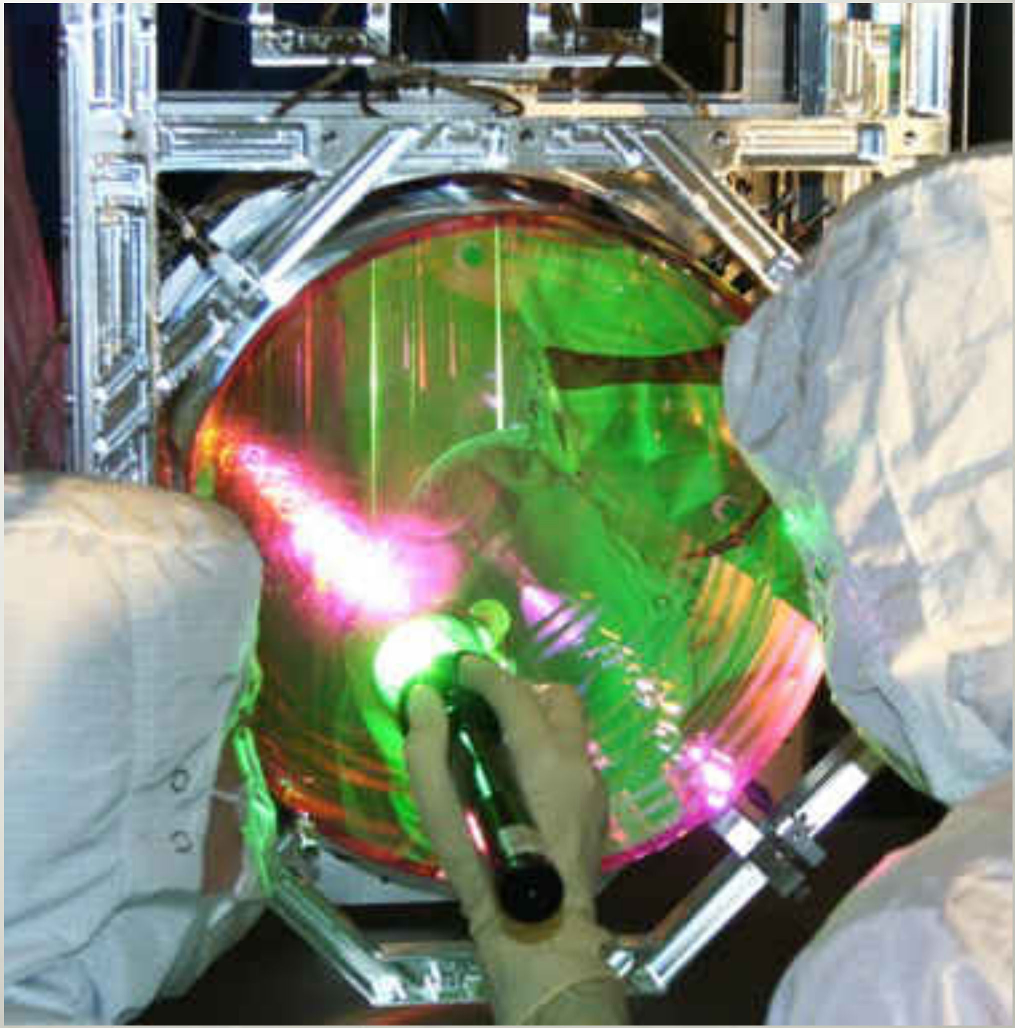
Experimental Techniques in GW Detection



Rana Adhikari Caltech

OUTLINE

- Gravitational Waves and History
- Some Noise Lingo
- Optics and Resonators
- Interferometers and Noise
- Electronics and Control Systems
- Advanced Topics



The Michelson Interferometer

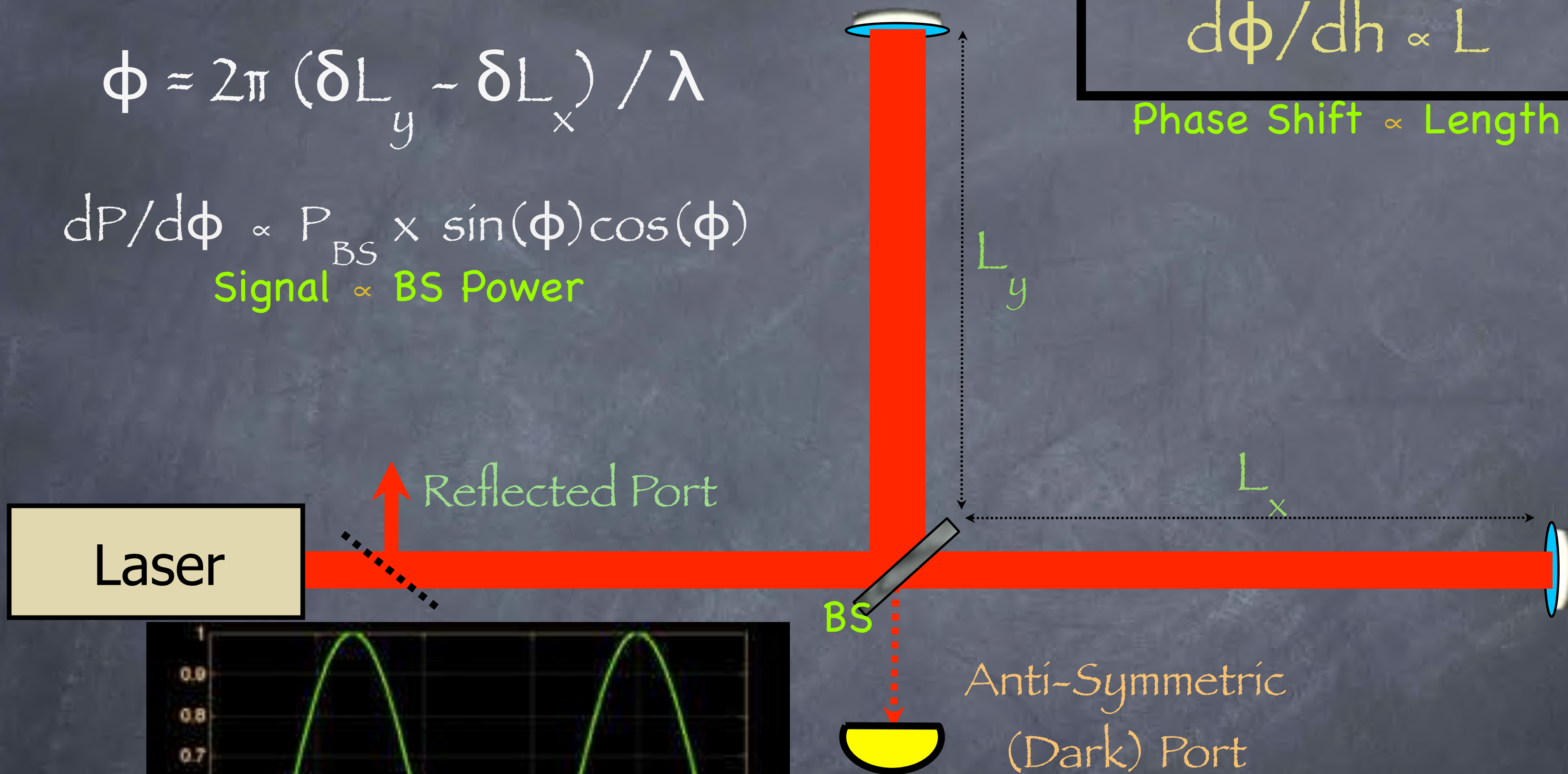
$\phi = 2\pi\,(\delta L_y - \delta L_x)\,/\,\lambda$

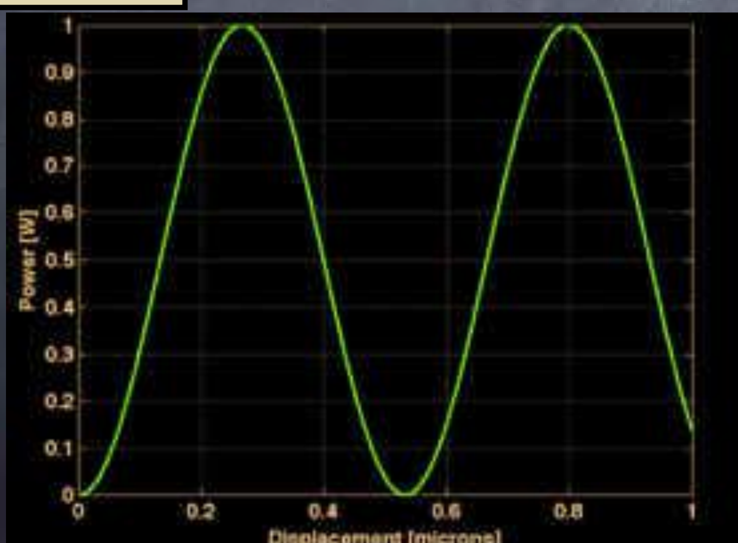
$d\phi/dh \propto L$

Phase Shift ∝ Length

$dP/d\phi \propto P_{BS} \times \sin(\phi)\cos(\phi)$

Signal ∝ BS Power



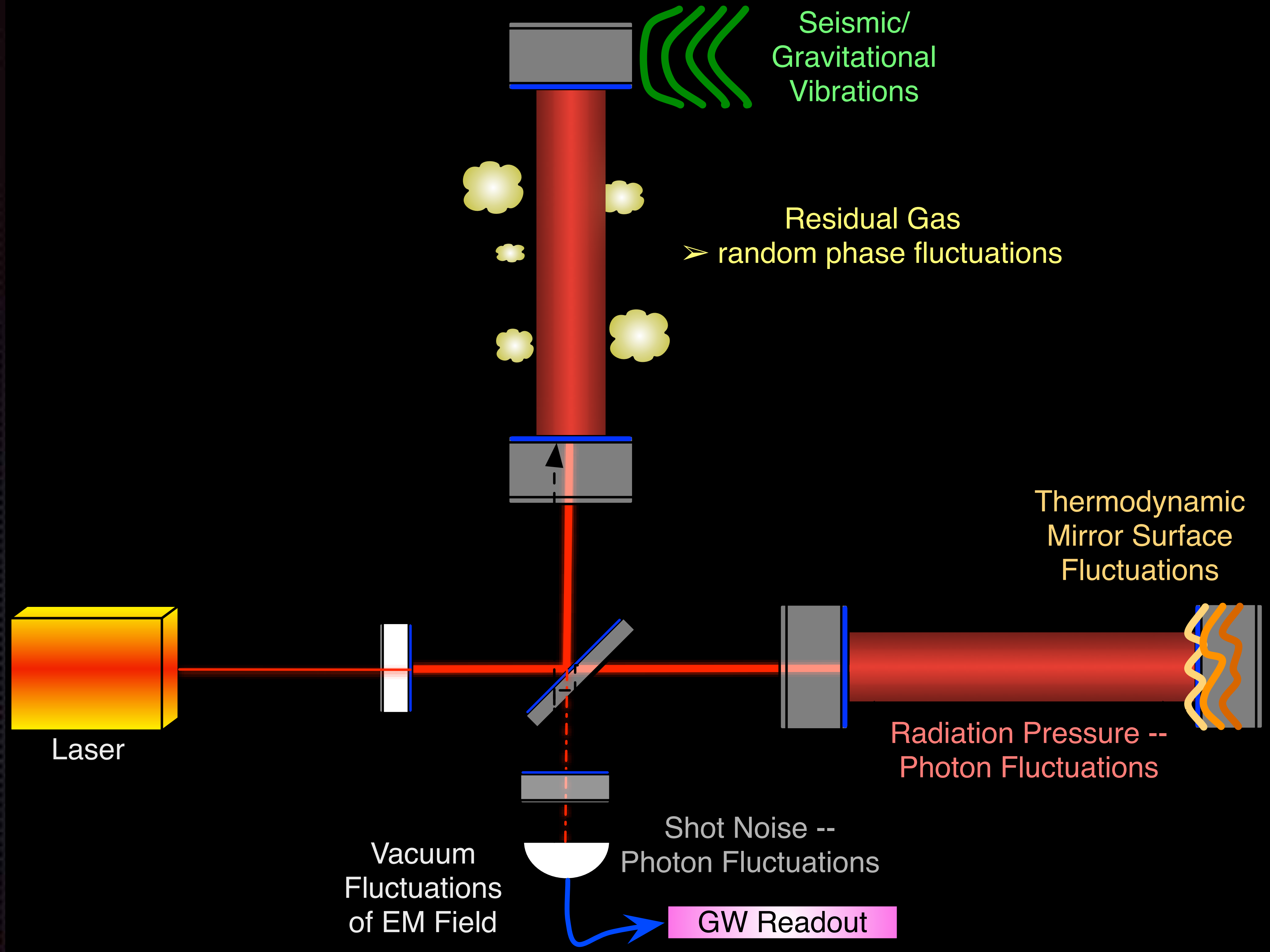


$P \propto P_{BS} \times \sin^2(\phi)$

Poisson Statistics...

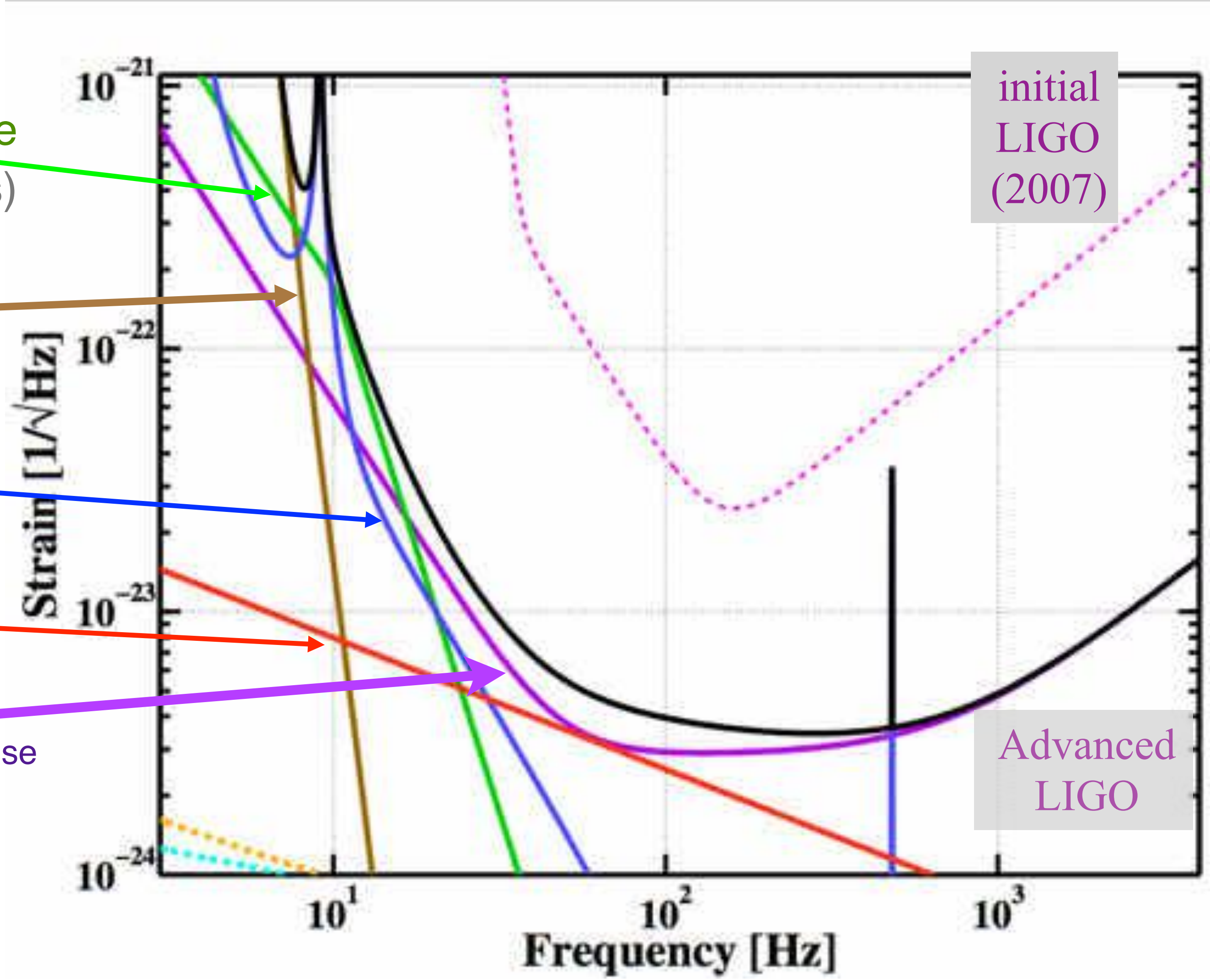
$dP \propto \mathrm{sqrt}(P)$

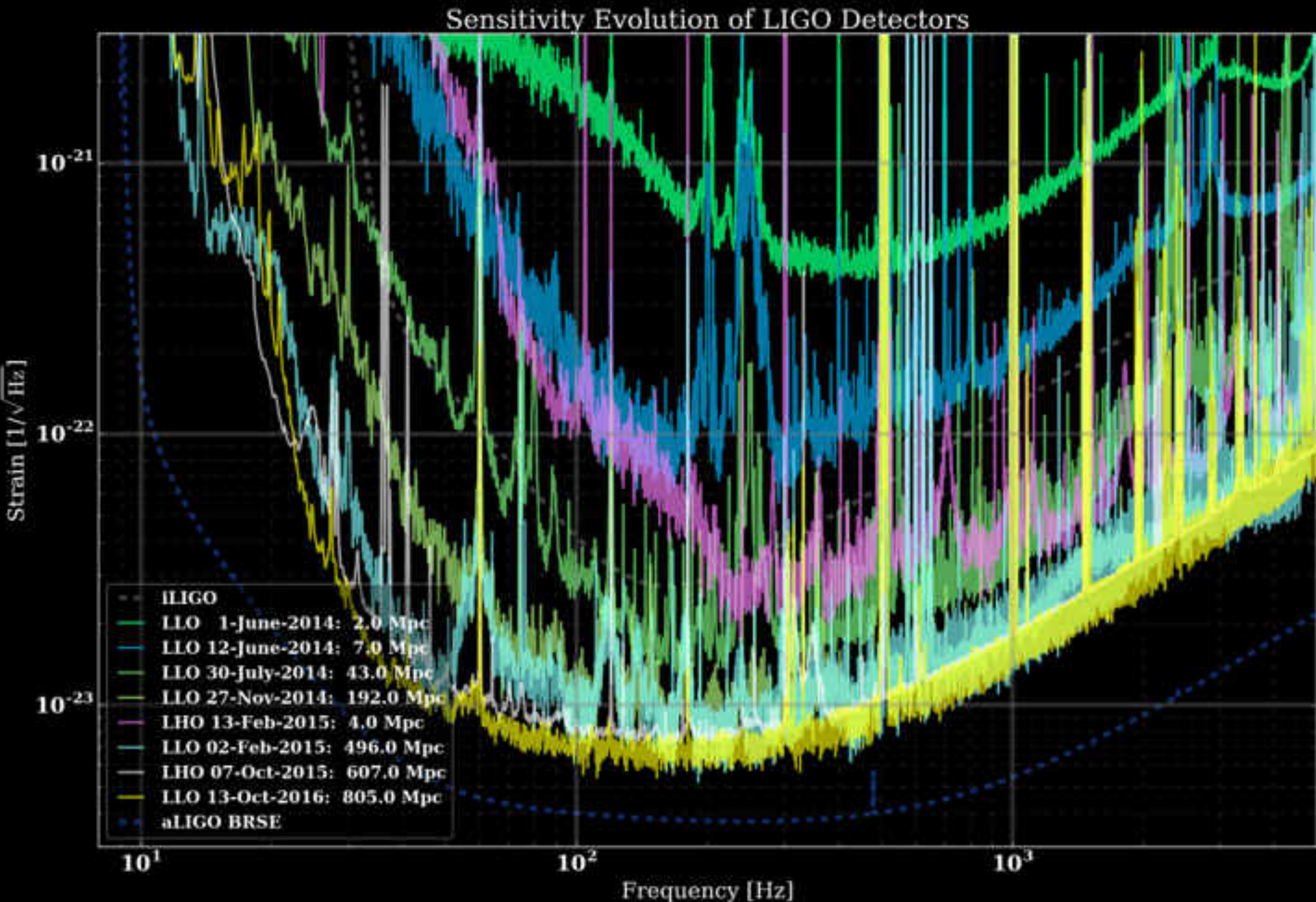
Shot Noise

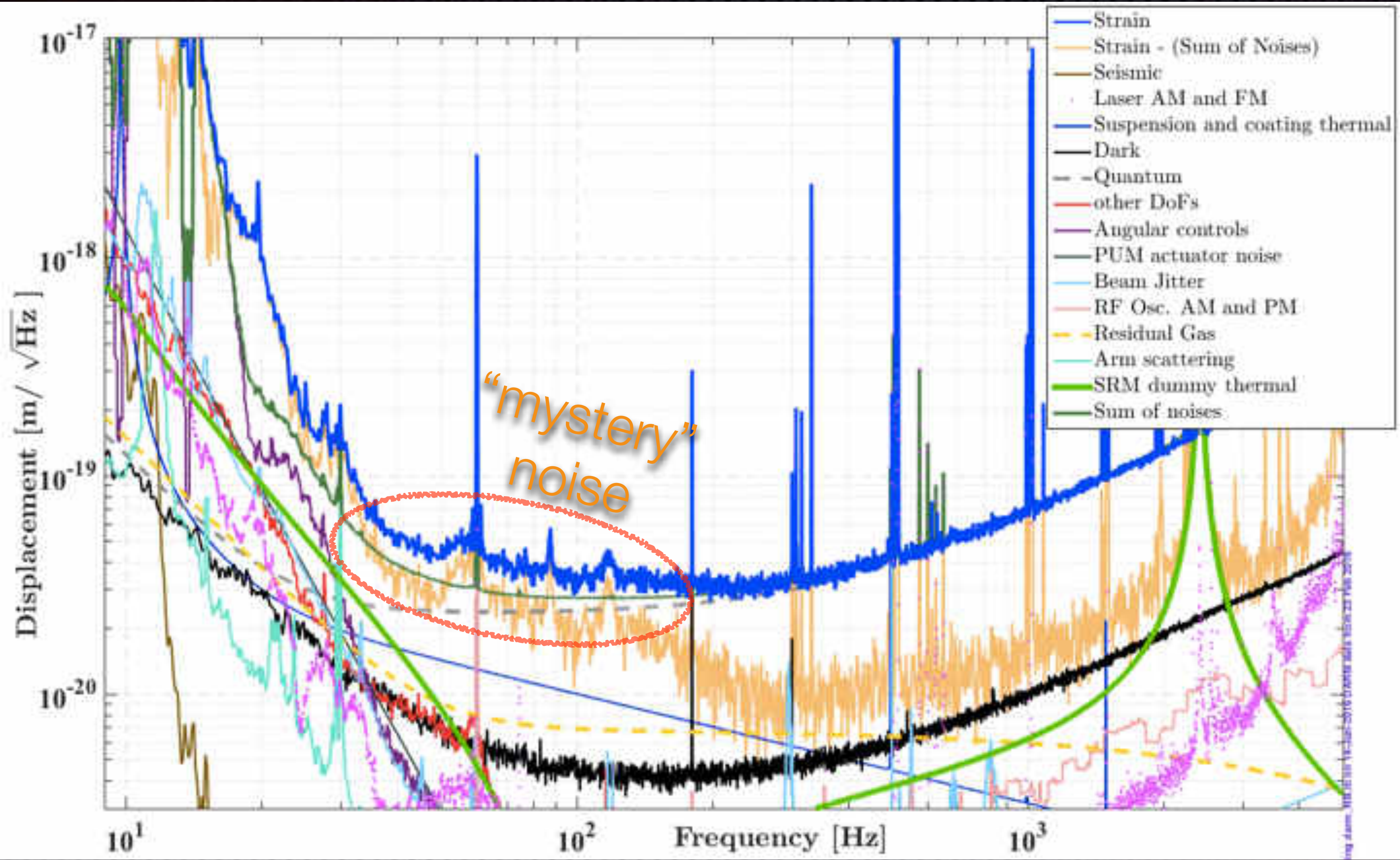

Seismic/
Gravitational
Vibrations
Residual Gas
➢ random phase fluctuations
Thermodynamic
Mirror Surface
Fluctuations
Laser
Radiation Pressure --
Photon Fluctuations
Shot Noise --
Photon Fluctuations
Vacuum
Fluctuations
of EM Field
GW Readout

Anatomy of the Interferometer Performance

- ☑ Newtonian gravity noise
  (a.k.a. Gravity Gradients)
- ☑ Filtered Seismic
- ☑ Silica Suspension Thermal Noise
- ☑ Mirror Coating Thermal
- ☑ Quantum Noise:
  Radiation Pressure / Shot Noise



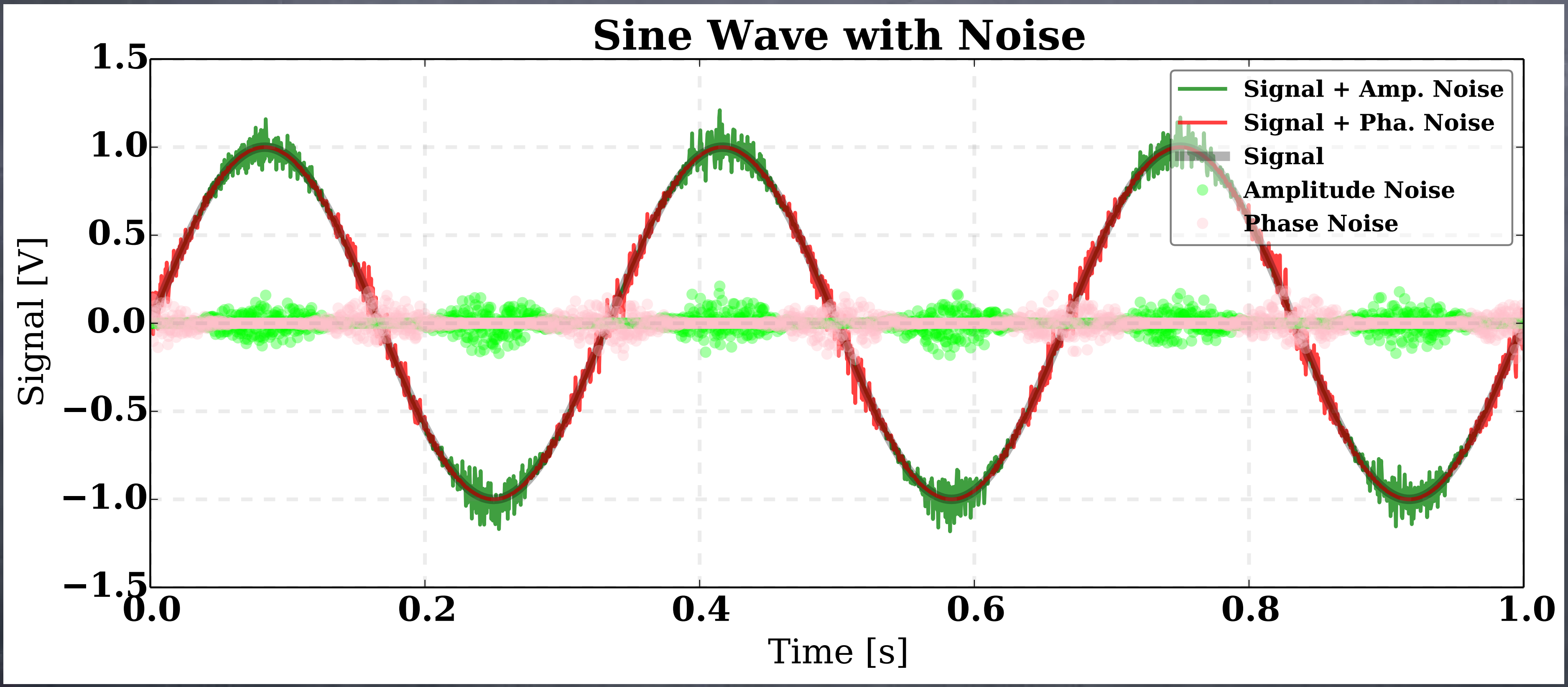

Sensitivity Evolution of LIGO Detectors
Strain [1/√Hz]
10^-21
10^-22
10^-23
10^1
10^2
10^3
Frequency [Hz]
iLIGO
LLO 1-June-2014: 2.0 Mpc
LLO 12-June-2014: 7.0 Mpc
LLO 30-July-2014: 43.0 Mpc
LLO 27-Nov-2014: 192.0 Mpc
LHO 13-Feb-2015: 4.0 Mpc
LLO 02-Feb-2015: 496.0 Mpc
LHO 07-Oct-2015: 607.0 Mpc
LLO 13-Oct-2016: 805.0 Mpc
aLIGO BRSE



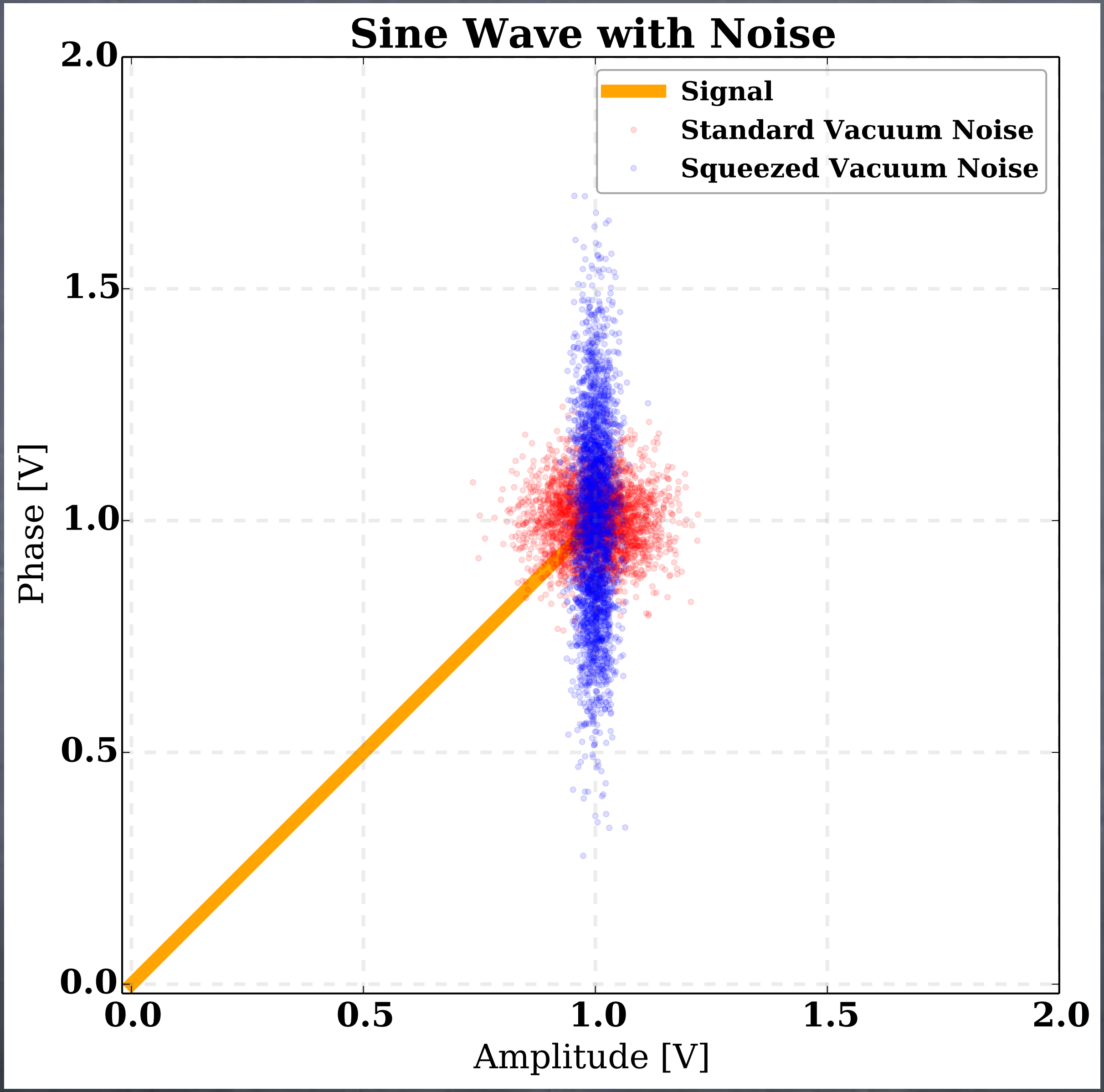
estimated Noise Budget (Louisiana, Feb. 2016)

Sources of Noise: Quantum

Noise Quadratures



$$\Delta E \Delta \phi \geq \hbar\omega/2$$

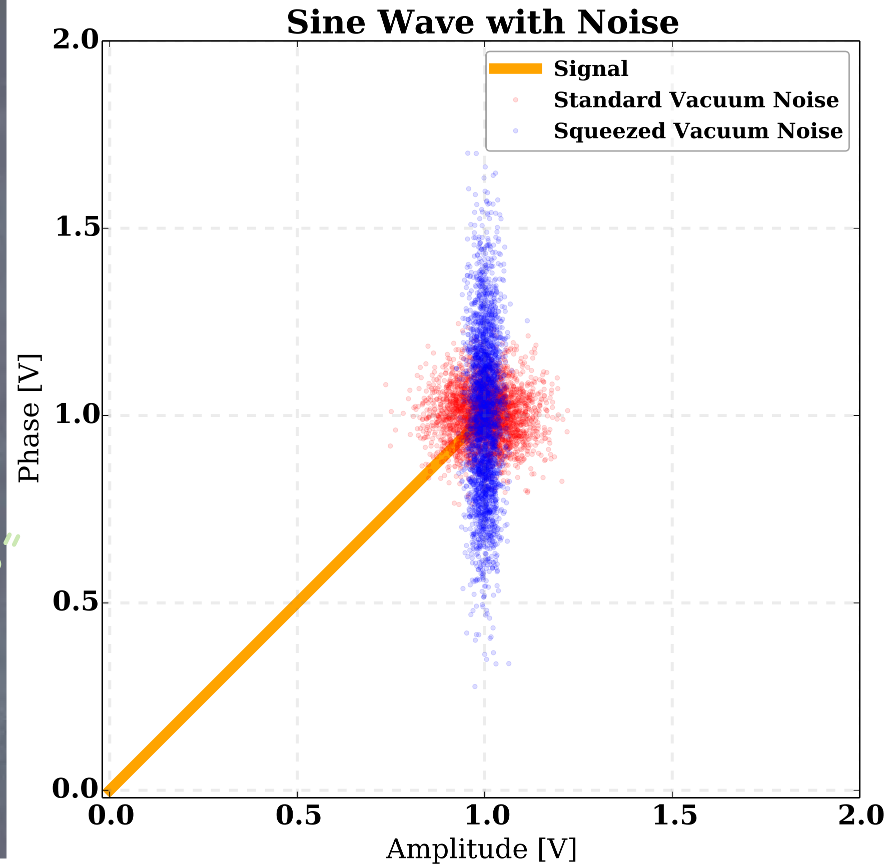


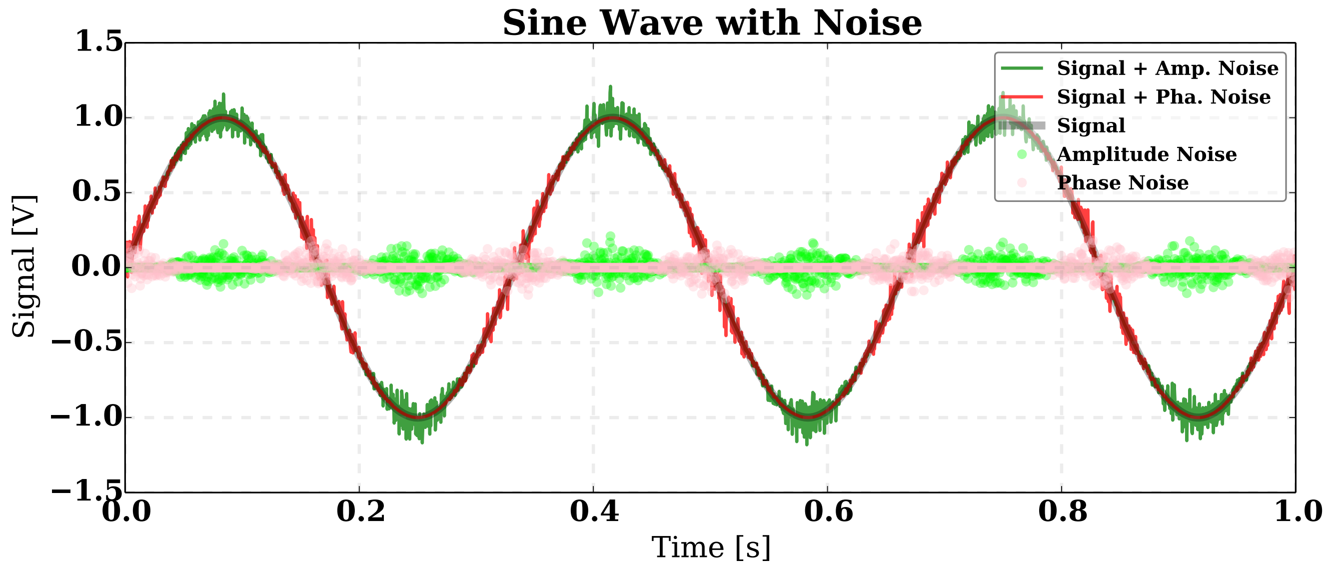
$$\Delta E \Delta \phi \geq \hbar\omega/2$$

Noise Quadratures

$$e^{i\theta} = \cos(\theta) + i\,\sin(\theta)$$

"the most remarkable formula in mathematics"
-R.P. Feynman





$$\Delta E \Delta \phi \geq \hbar\omega/2$$

Useful Optics for Laser Interferometers

Optics and Resonators

- Optics 101 -> Ray Matrices
- Gaussian Beams, Gaussian Beam Propagation
- Fabry-Perot Cavities
- Higher Order Transverse Modes
- Simple Cavity Locking (Why PDH?)

Ray Matrices

(geometric optics)

M

$$\begin{pmatrix} x_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 0 \\ -2/RoC & 1 \end{pmatrix}$$

$$\vec{x}_2 = P_{23} L_2 P_{12} L_1 P_{01} \vec{x}_1$$

- A. E. Siegman, “Lasers”
- Wikipedia – http://en.wikipedia.org/wiki/Ray_transfer_matrix_analysis
- A. Yariv, “Quantum Electronics”

Gaussian Beams

(slightly beyond geometric optics)

$$E(r,z) = E_0 \frac{w_0}{w(z)} \exp\left(\frac{-r^2}{w^2(z)} - ikz - ik\frac{r^2}{2R(z)} + i\zeta(z)\right)$$

Gaussian

Spherical RoC

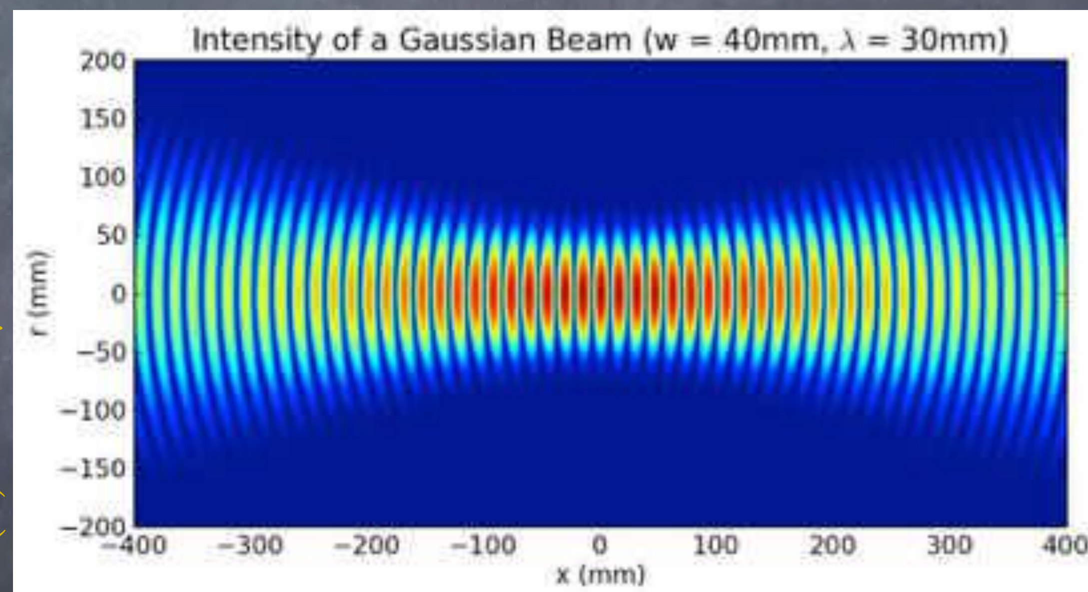
$$\zeta(z) = atan(\frac{z}{z_R})$$

Gouy Phase

Eigenmodes of a two spherical mirror optical resonator

$$R(z) = z(1 + (z_R/z)^2) \qquad z_R = \frac{\pi \omega_0^2}{\lambda}$$

$$\omega(z) = \omega_0 \sqrt{1 + (z/z_R)^2}$$



Etoombs at en.wikipedia

- A. E. Siegman, “Lasers”
- Wikipedia - http://en.wikipedia.org/wiki/Gaussian_beam
- A. Yariv, “Quantum Electronics”

Gaussian Beams

(w/ Ray Matrices)

$$P = \begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$$

$$\omega(z) = \omega_0 \sqrt{1 + (z/z_R)^2} \quad R(z) = z(1 + (z_R/z)^2)$$

$$L = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

$$z_R = \frac{\pi \omega_0^2}{\lambda}$$

$$M = \begin{pmatrix} 1 & 0 \\ -2/RoC & 1 \end{pmatrix}$$

$$q(z) = z + q_0 = z + i z_R$$

$$q_2 = \frac{A q_1 + B}{C q_1 + D}$$

- A. E. Siegman, “Lasers”
- Wikipedia – http://en.wikipedia.org/wiki/Gaussian_beam
- A. Yariv, “Quantum Electronics”

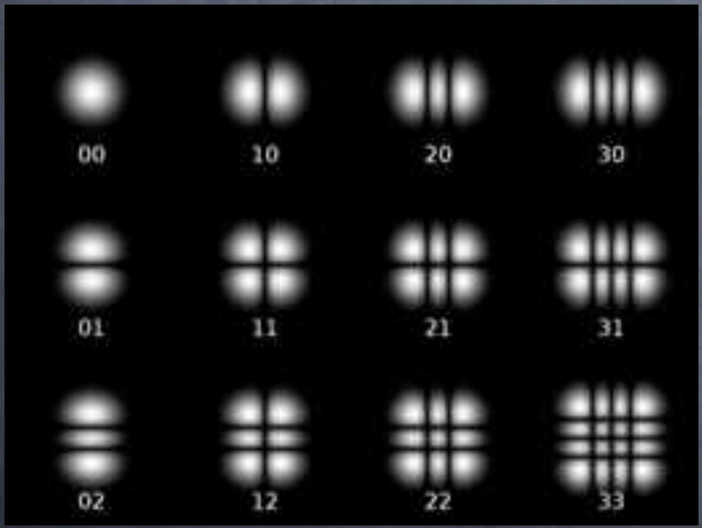
Higher - Order Modes

$$z_R = \frac{\pi\omega_0^2}{\lambda}$$

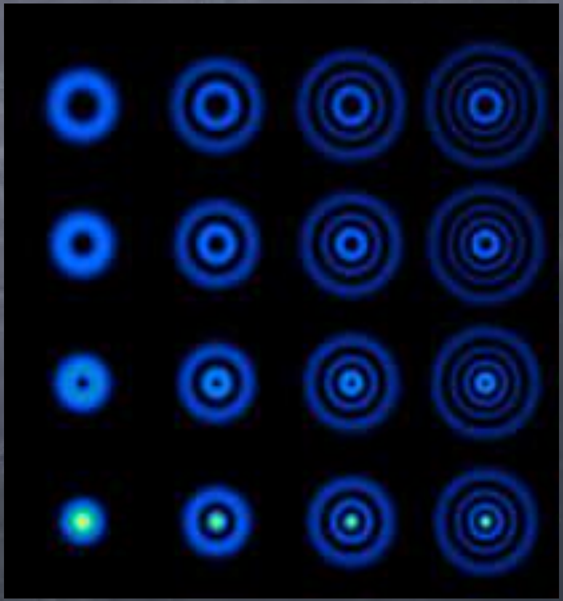
$$\omega(z) = \omega_0\sqrt{1+(z/z_R)^2}$$

$$u_{mn}(x,y,z) = \frac{z_R}{z+iz_R}\sqrt{\frac{2}{2^{m+n}m!n!\lambda}}H_m(\frac{\sqrt{2}x}{w(z)})H_n(\frac{\sqrt{2}y}{w(z)})$$

$$\times exp[-i\frac{x^2+y^2}{z+iz_R}\frac{k}{2}+i(m+n+1)arctan(\frac{z}{z_R})]$$



Hermite-Gaussian Modes



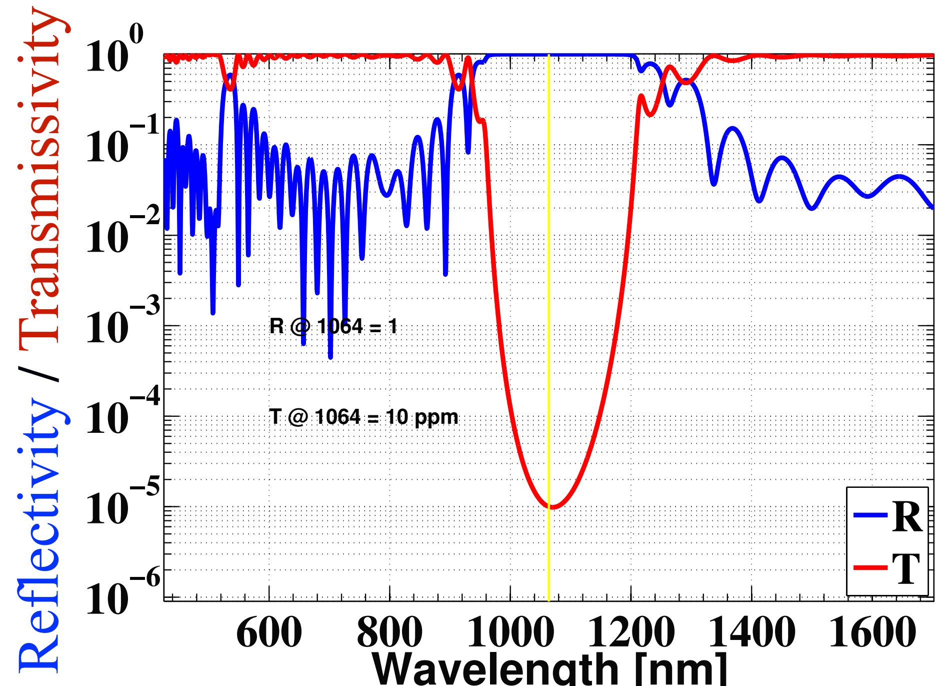
Laguerre-Gaussian Modes

- A. E. Siegman, "Lasers"
- Wikipedia - http://en.wikipedia.org/wiki/Gaussian_beam
- A. Yariv, "Quantum Electronics"

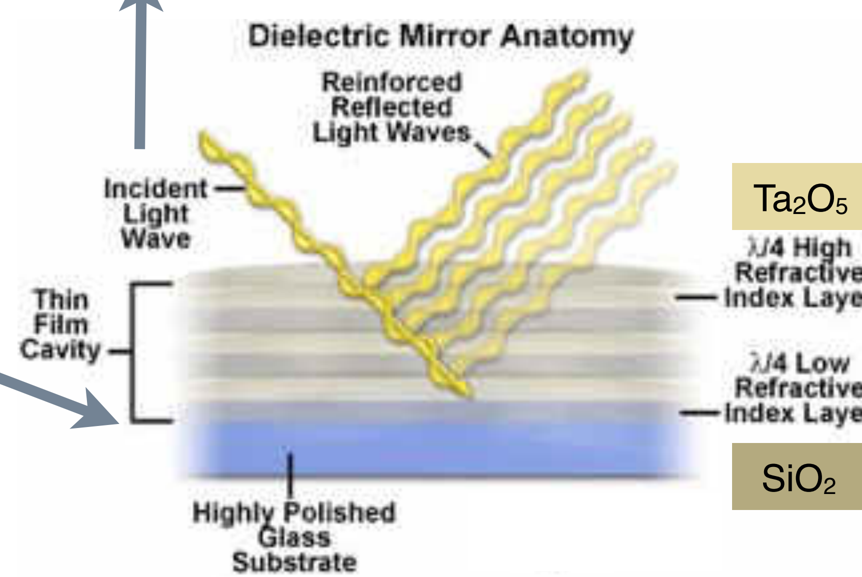
Making of a Mirror

Inertial Guidance

Litton (1979)
Ring Laser Gyro



Ion Beam Sputtered
Mirror Coatings

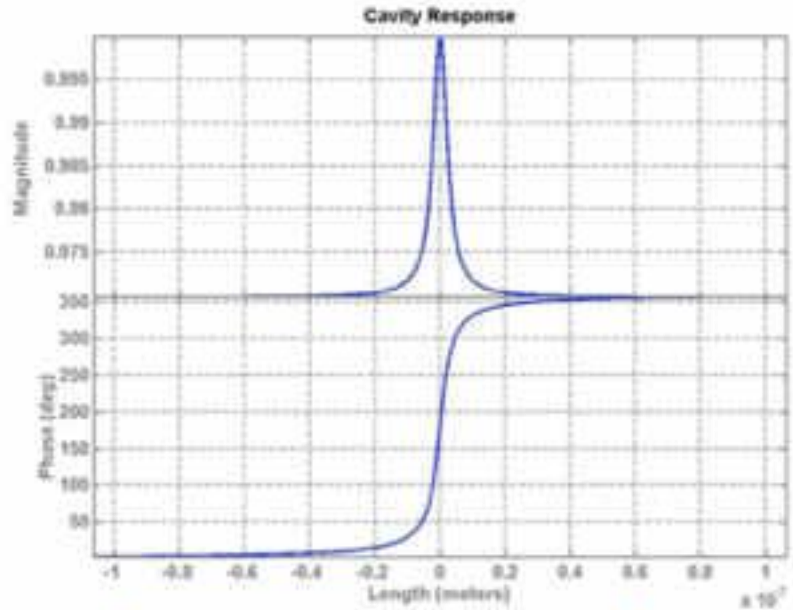


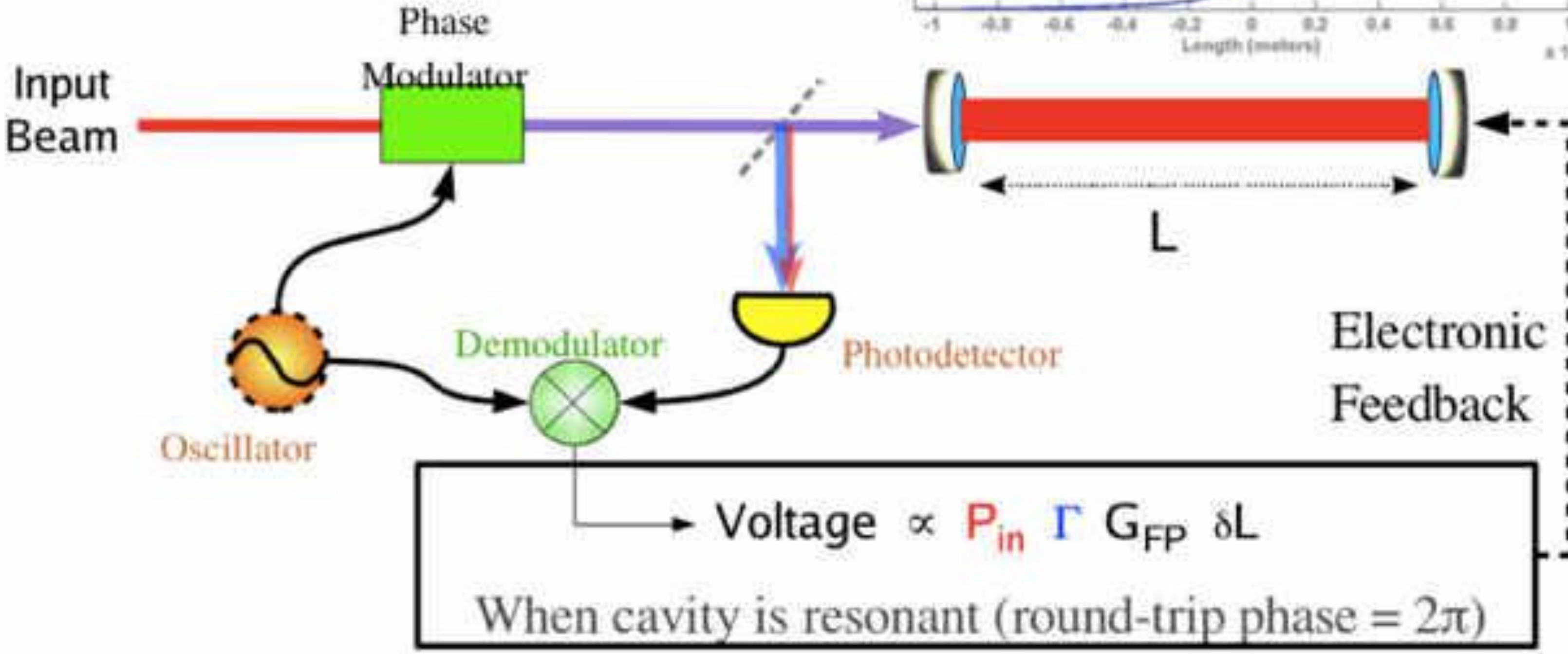
Locking an Arm Cavity

Phase Modulation =>

$$E = E_{in} \times e^{i\,2\Gamma \cos(\omega_m t)}$$

$$\cong E_{in} \times [1 + i\,\Gamma e^{i\,\omega_m t} + i\,\Gamma e^{-i\,\omega_m t}]$$





Thermodynamic Noise

Thermal Noise of a Mirror

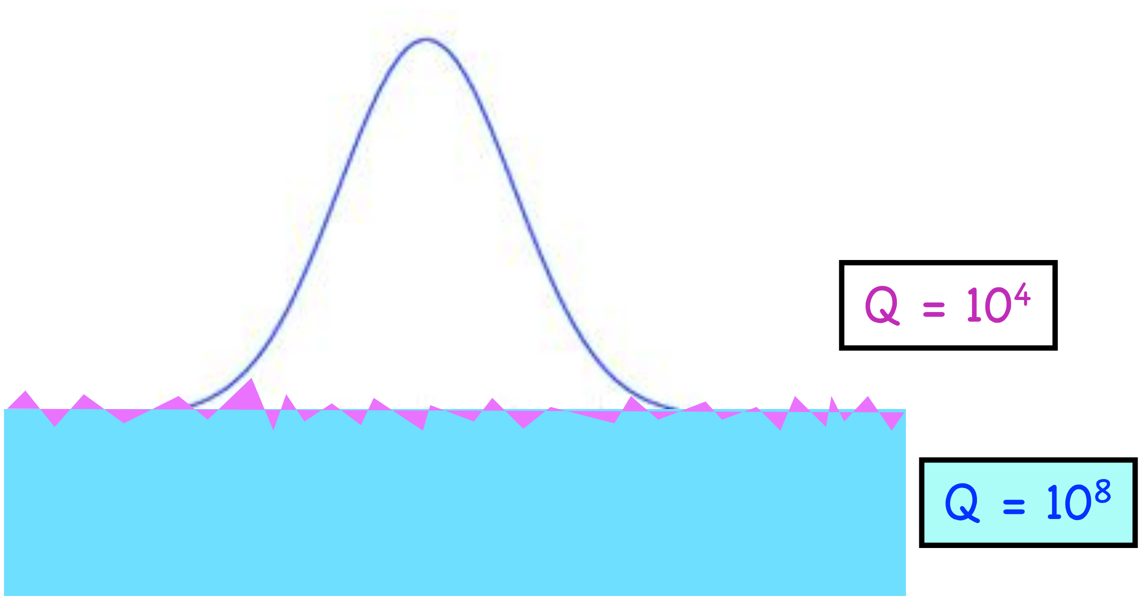
$$S_x(\omega) = \frac{4k_B T}{\omega^2 Re[Z(\omega)]}$$

Fluctuation-Dissipation Theorem

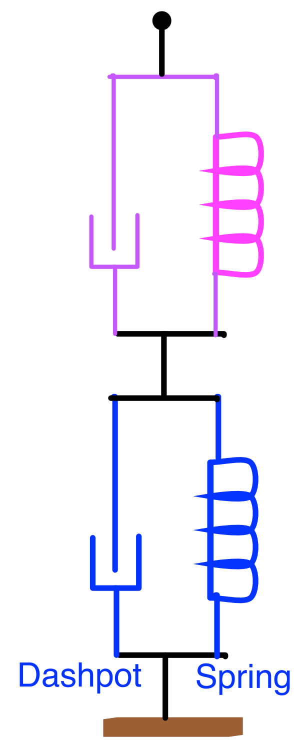
Callen and Welton, Phys. Rev. (1951)

$$Z(\omega) = m\frac{\omega_0^2 - i\omega_0\omega/Q - \omega^2}{i\omega}$$

Single Damped Harmonic Oscillator



Mirror Surface
Thermal Fluctuations

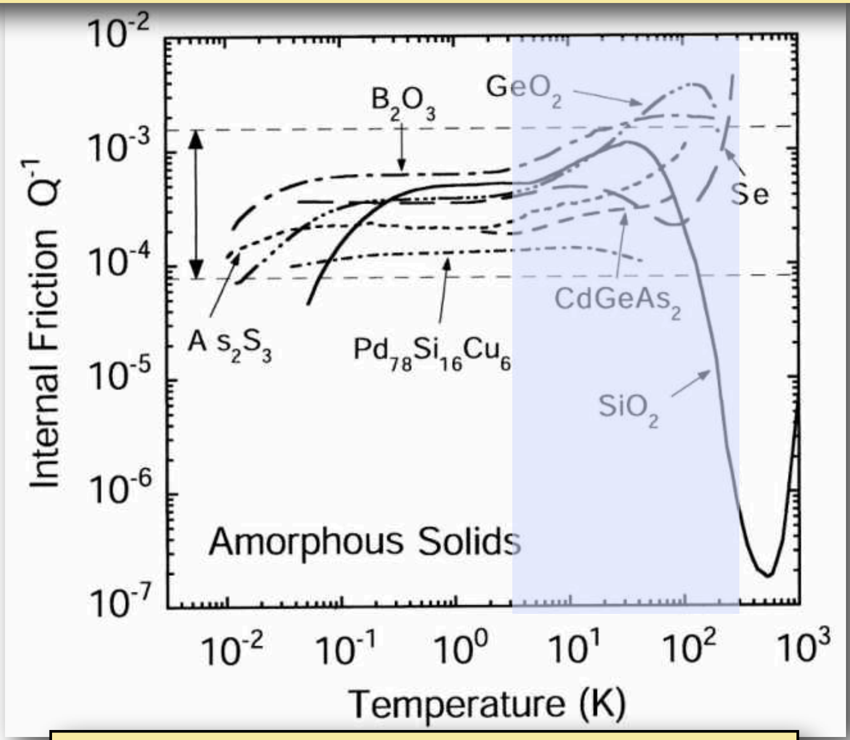


Yuri Levin, PRD (1998)

Hong, Yang, Gustafson, **RA**, Chen. PRD (2013)

Thermal Noise of a Mirror

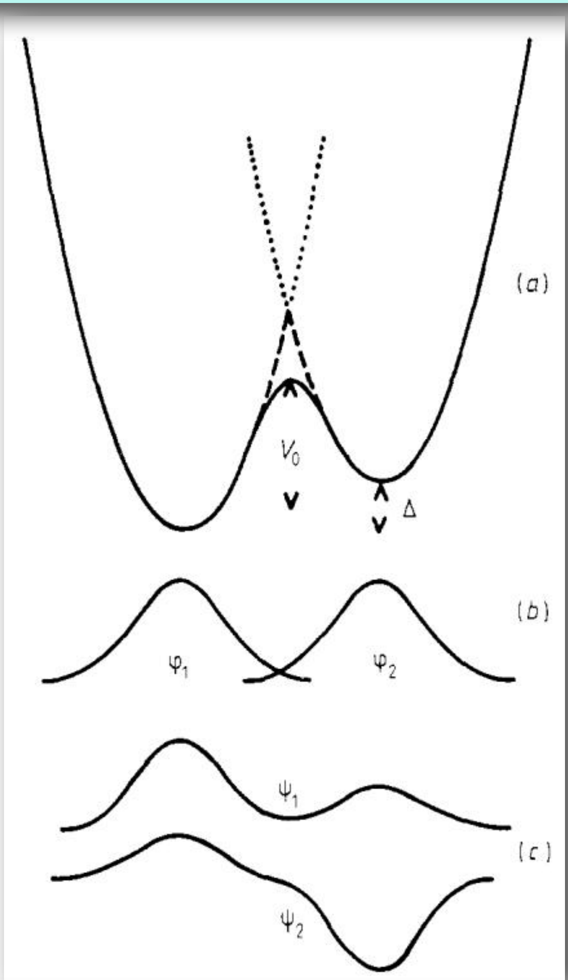
Why a ratio of 10^4 in dissipation?



R.O. Pohl, et al., Rev. Mod. Phys. (2002)

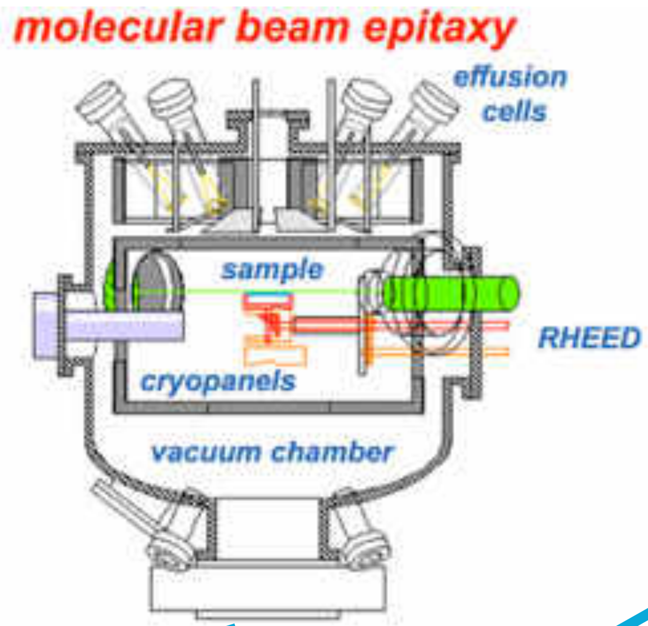
- ***Nearly all high quality optical coatings use amorphous oxides.***
- ***Nearly all amorphous materials have a (low Q) large internal friction.***

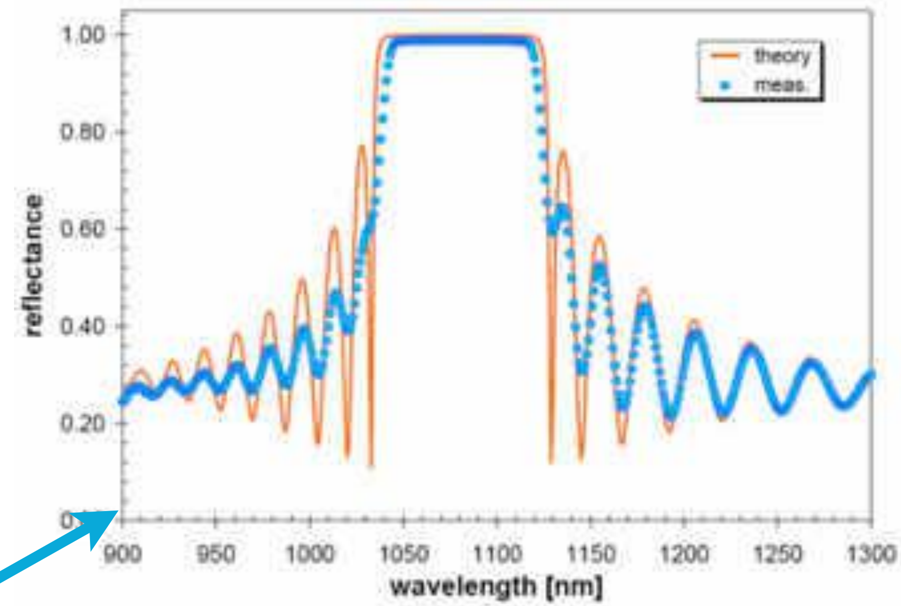
2-level tunneling model

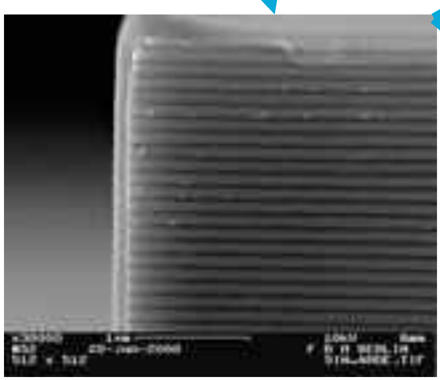


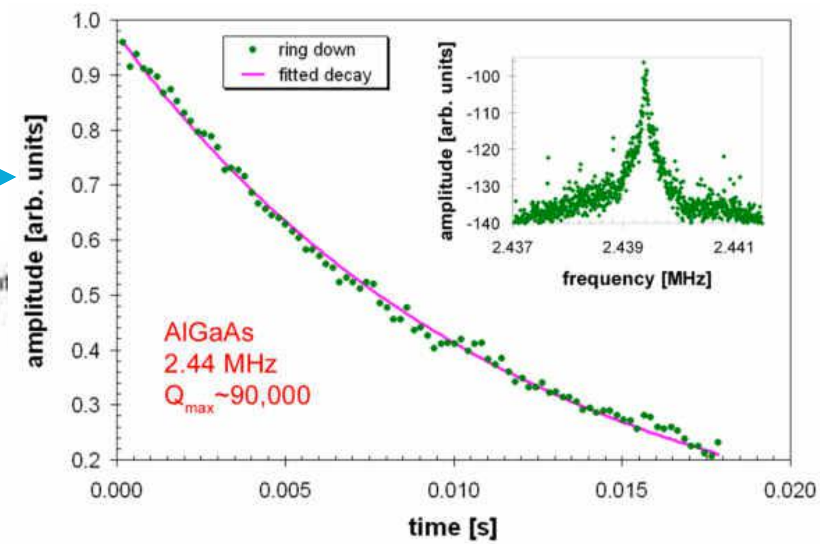
W.A. Phillips, Rep. Prog. Phys. (1987)

Crystalline Mirror Coatings





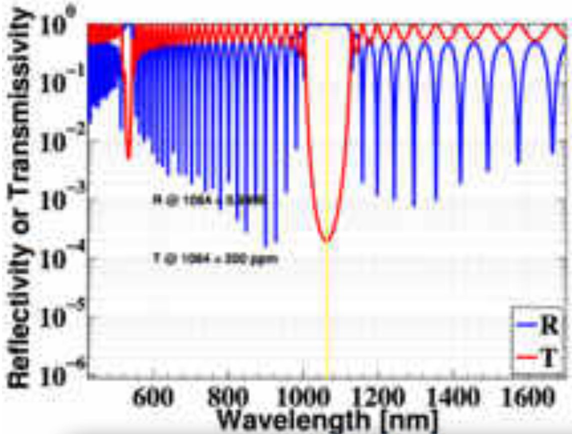




G. D. Cole, et al., Appl. Phys. Lett. (2010)

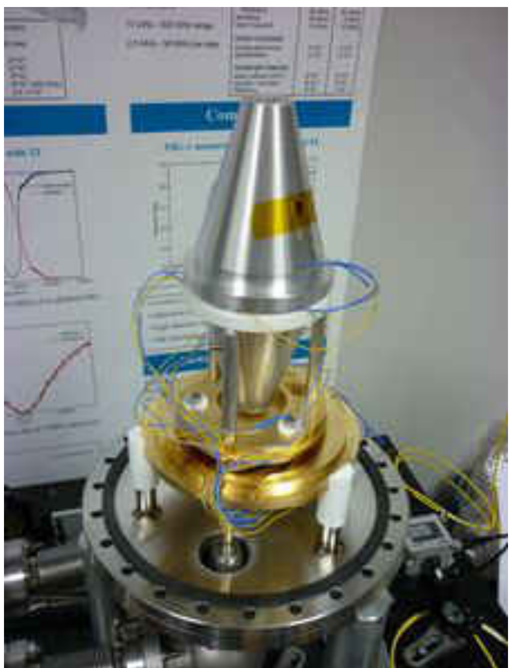
$$Q_{coating} \sim 10^5 - 10^6$$

The Road to Noiseless Mirrors



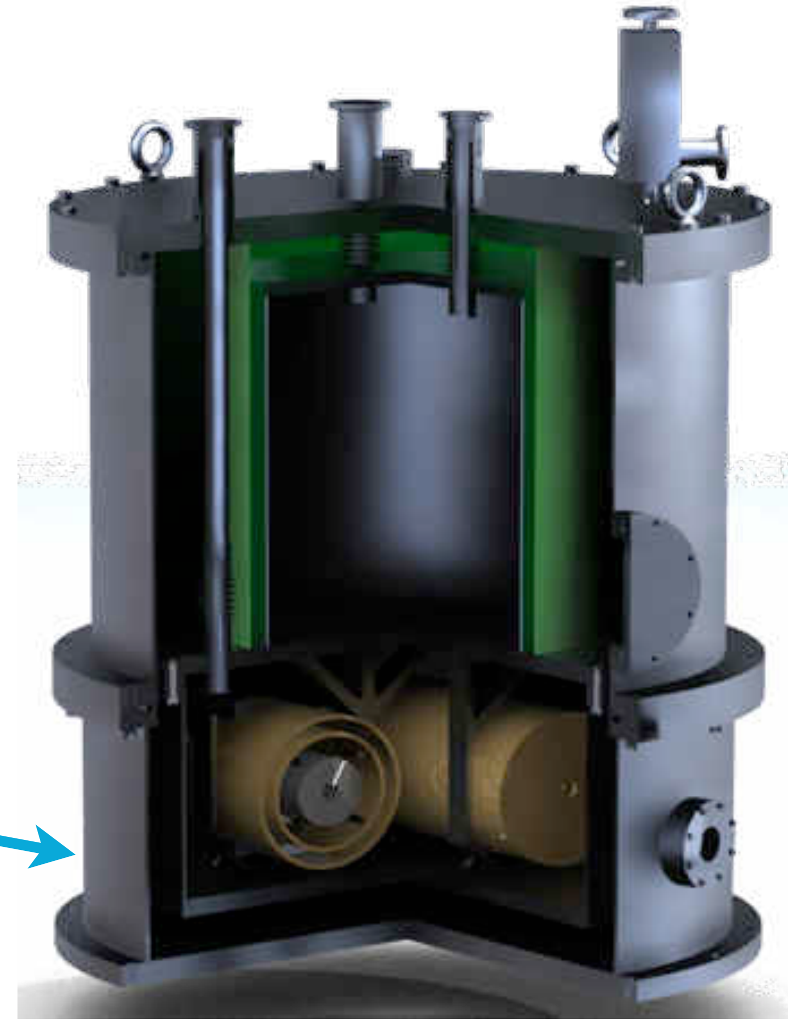
300K design

*G. D. Cole, **RA**, F. Seifert, in prep. (2012)*



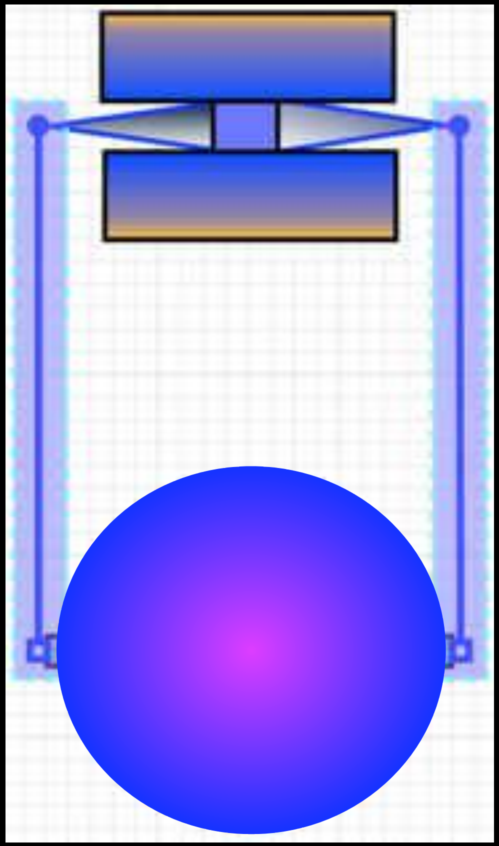
JILA / PTB

120K Silicon:
CTE = zero,
High Thermal
Conductivity



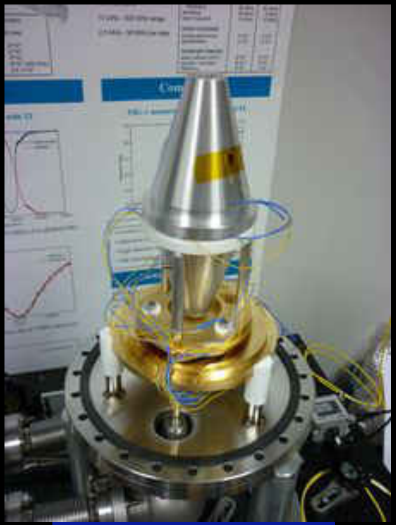
Caltech IQIM

Cryogenic LIGO



Monolithic Silicon Suspension

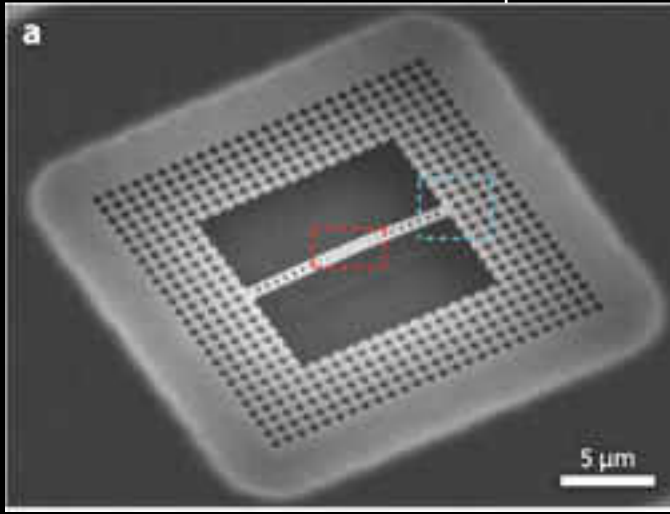
Caltech IQIM



JILA / PTB

Silicon etch process from O. Painter @ Caltech for ground state cooling

f ~4 GHz, Q ~10^5, $N_{quanta} < 1$



Requires switching the laser technology:
1064 nm => 1550 nm

Quantum Noise

A Quantum Interlude:

wherein we ask the question

"what is the hard limit to spacetime sensing??"

Why a Quantum limit?

- The GW is nearly **noiseless**; spacetime strain should be compared with Planck-scale fluctuations. **SNR > 10^{13}**
- In the audio band, technical noise (gravity, thermodynamics, gas scattering, laser noise) can be mitigated*.
- Photon shot noise is not fundamental.
- The Heisenberg uncertainty on a 40 kg test mass is near the Planck scale.

* using sufficiently advanced technology

Towards the Fundamental Limit for Classical Force Sensing:

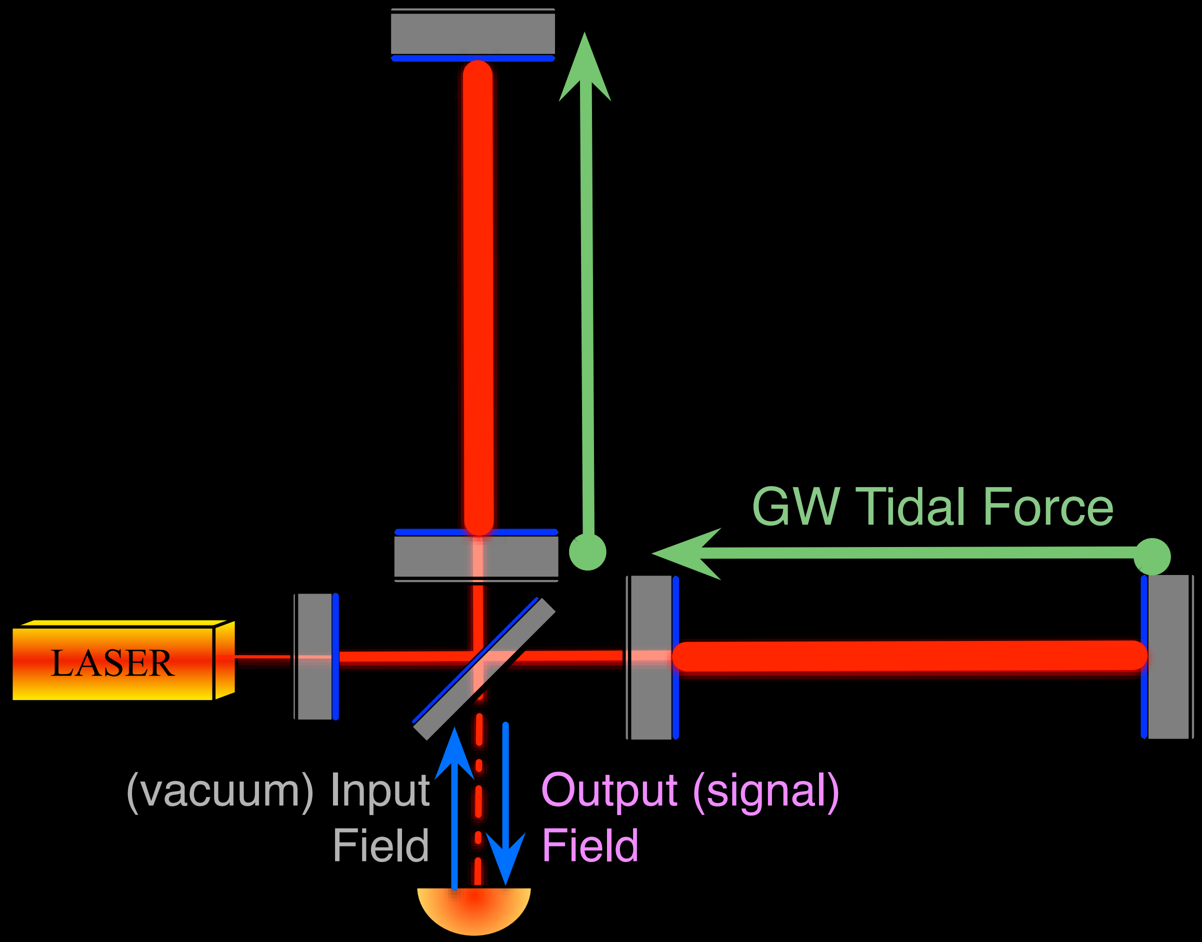
the Quantum Cramer-**Rao** bound

https://arxiv.org/abs/1608.00766

- **Gravitational wave: a Classical tidal force**
- the mirror is a quantum harmonic oscillator
- **Optical Field: Coherent state**
  =====> ***quantum fluctuations***

H. Miao, RXA, Belinda Pang, Yiqui Ma,Yanbei Chen

Quantum model of a GW Detector



Model: continuous Quantum measurement

Mirror: a quantum harmonic oscillator (QHO)

G Wave: a *Classical* force

Laser Field: a quantum field

Photo Diode: Projective Measurement

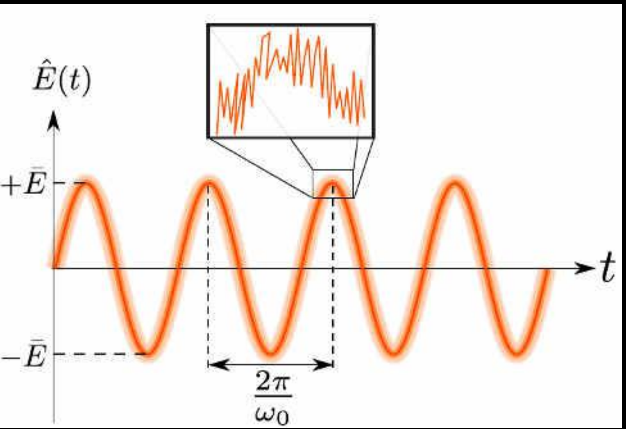
Quantum model of a GW Detector

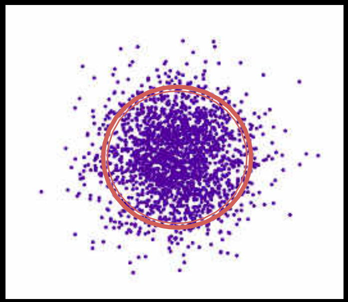
Quantization of Test Mass: $\frac{\omega_{CM}}{2\pi} \simeq 0.5Hz$

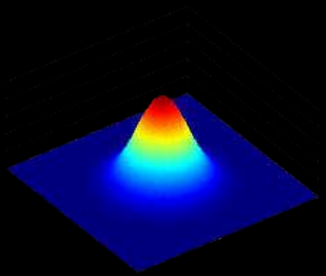
Mirror CoM motion: $k_B T/\hbar\omega_{CM} \simeq 10^{13}$

Quantization of Optical Field: $\frac{\omega_{opt}}{2\pi} \simeq 10^{13} Hz$

$k_B T/\hbar\omega_{opt} \simeq 0.02$

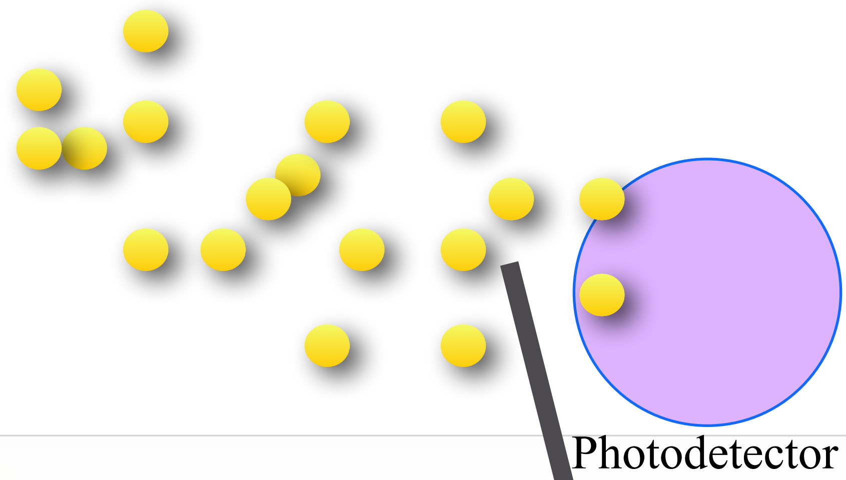




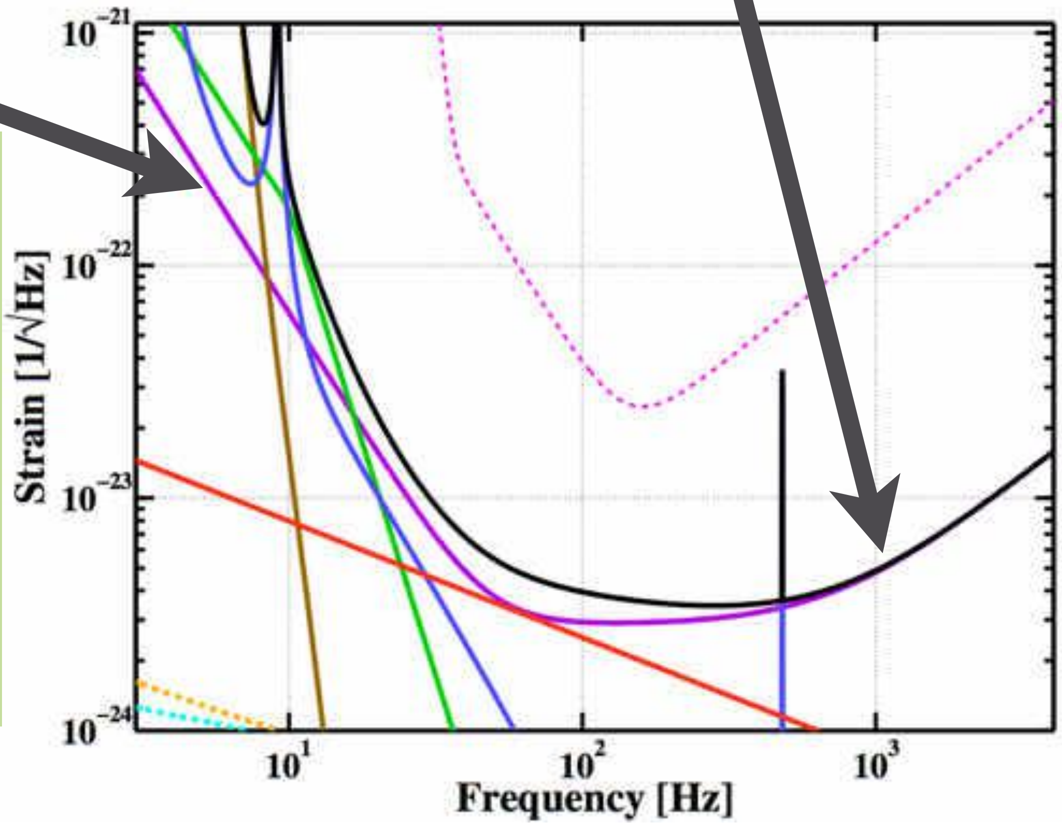


What about this Quantum noise?

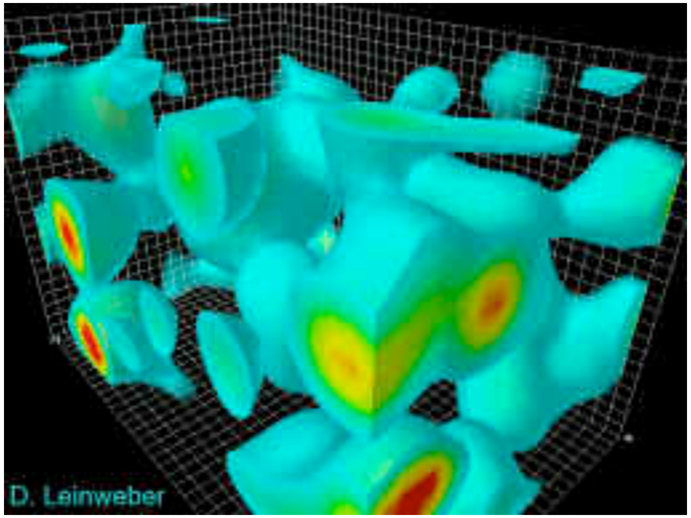
Shot Noise Picture:
Poisson statistics govern arrival time of photons at the photodetector. Also arrival times at the test mass (radiation pressure).

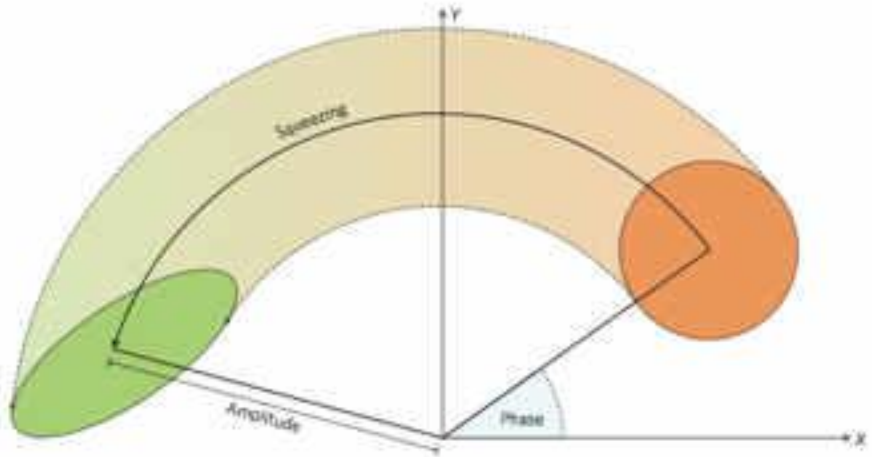


Vacuum Photon Picture:
Losses couple the fluctuating vacuum field to the interferometer. Noise is a beat between the amplitude of the vacuum field and the local field (field at the dark port or field at the test mass).

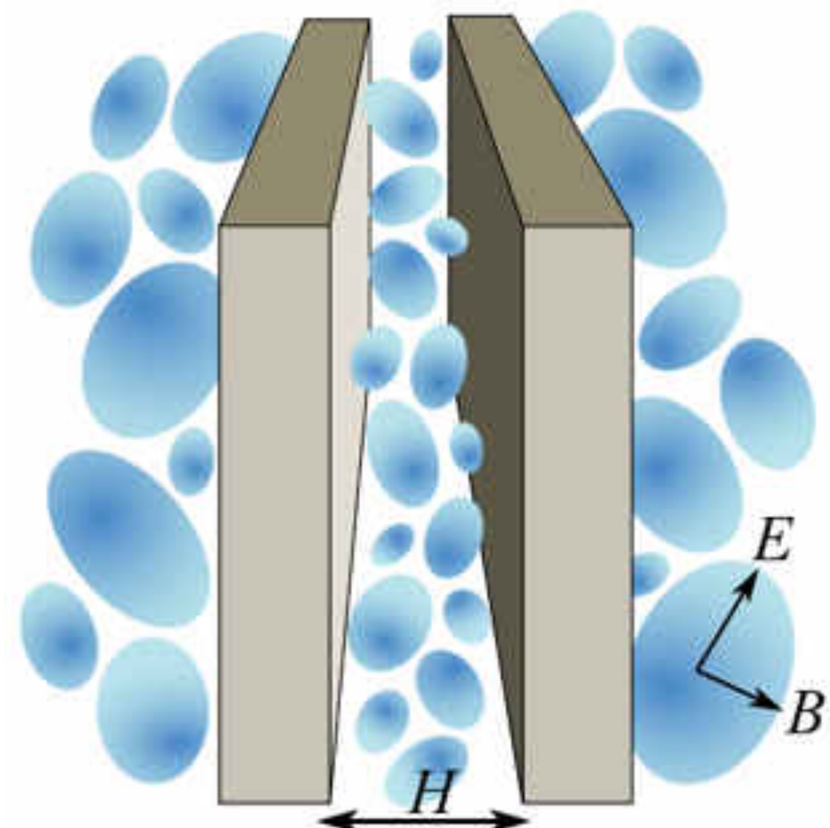


Vacuum State Fluctuations





Warwick Bowen, Nature Photonics (2013)



http://www.mit.edu/~kardar/research/seminars/Casimir2010/talks/Francqui/Casimir.html

[1] C. M. Caves, Physical Review A **31**, 3068 (1985).

[2] H. Miao and Y. Chen, “Adv. Grav. Wave Detectors” (2010)

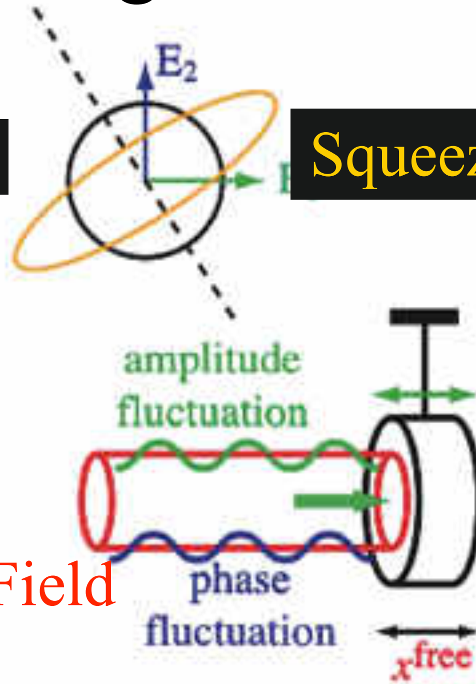
V. Braginsky (MSU) Kip Thorne

Caltech TAPIR group (1974)

Carleton Caves David Lee

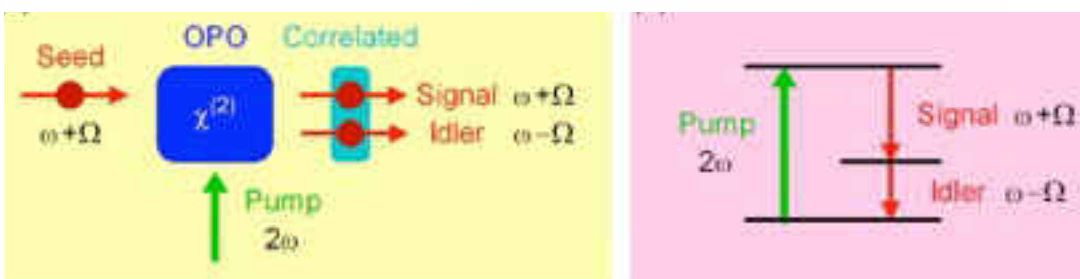
Circumventing Usual Quantum Noise

Vacuum



Squeezed Vacuum

Carrier Field



Atomic Polarization of a Dielectric Medium

$$P = \varepsilon_0(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \ldots)$$

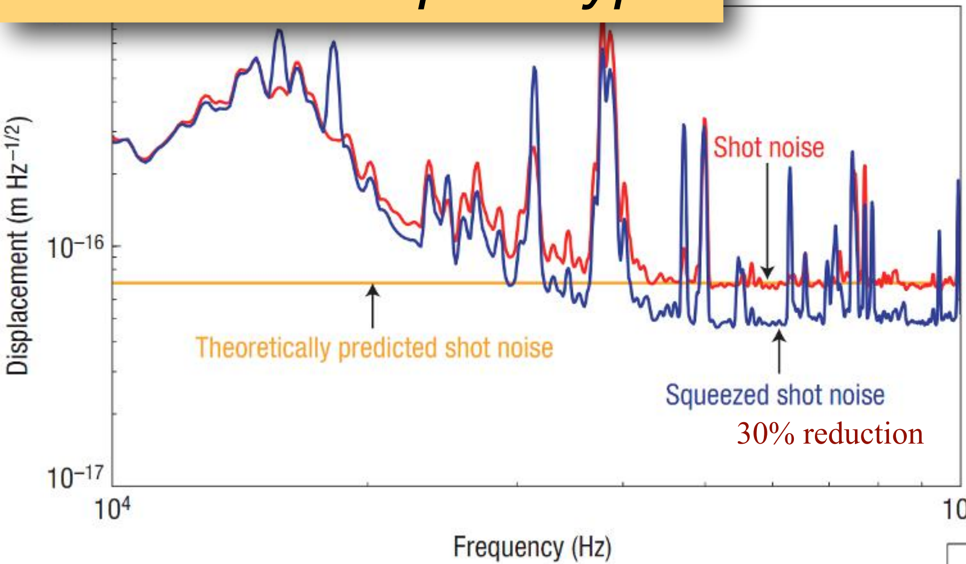
Braginsky, Vorontsov and Thorne, Science **(1980)**

C. M. Caves, PRD (1981)

Wu, Kimble, Hall, Wu, PRL (1986)

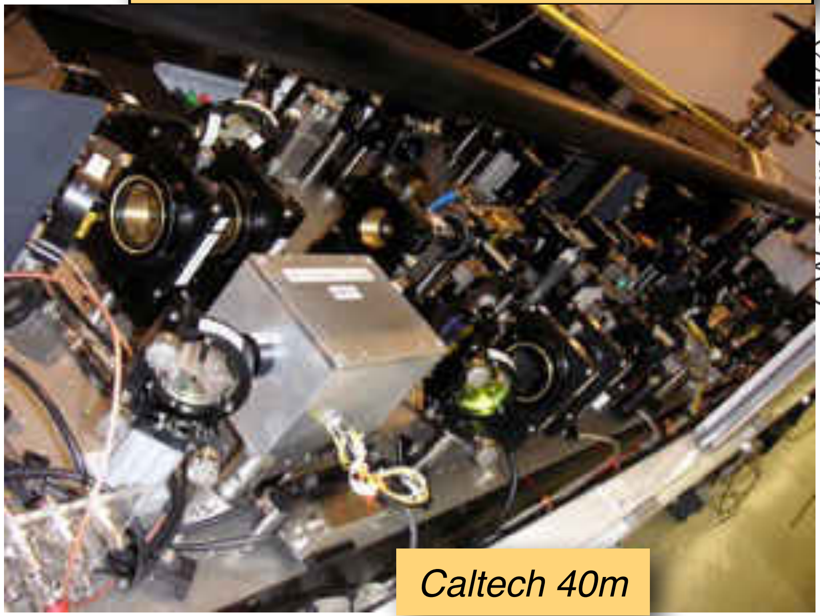
Squeezed Interferometers

Caltech 40m prototype

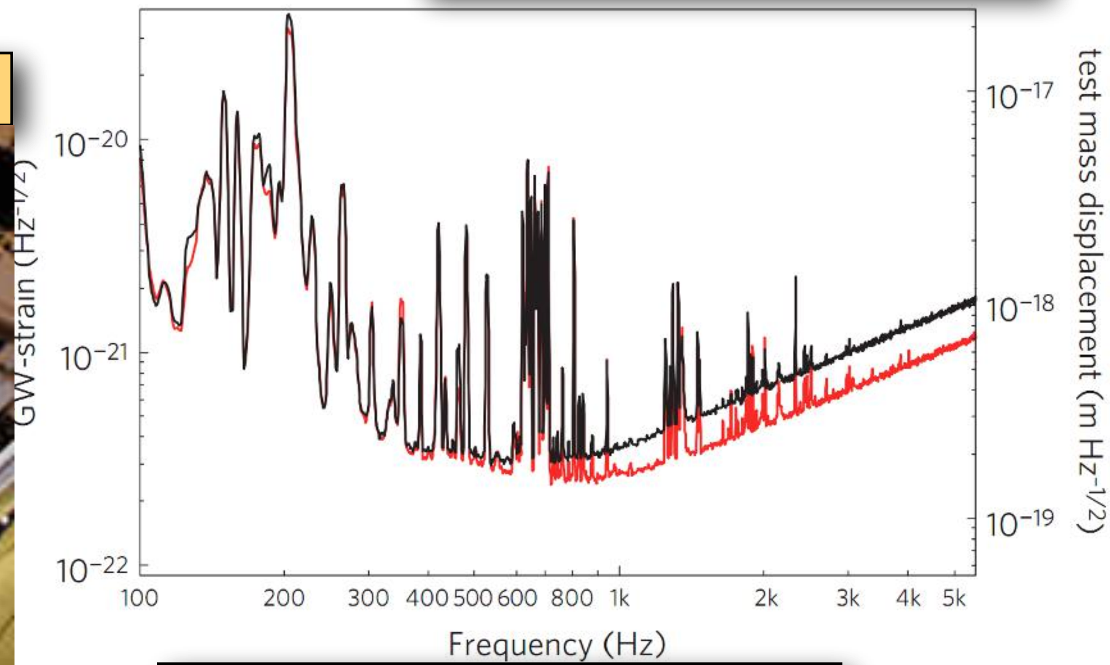


K. Goda, et al., Nature Physics (2008)

GEO Squeezer (Hannover)

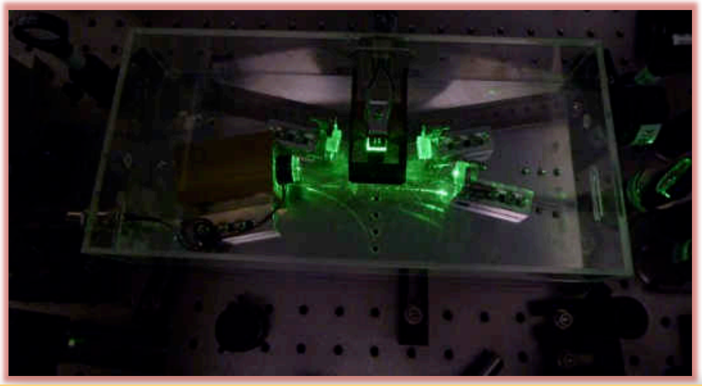


Caltech 40m



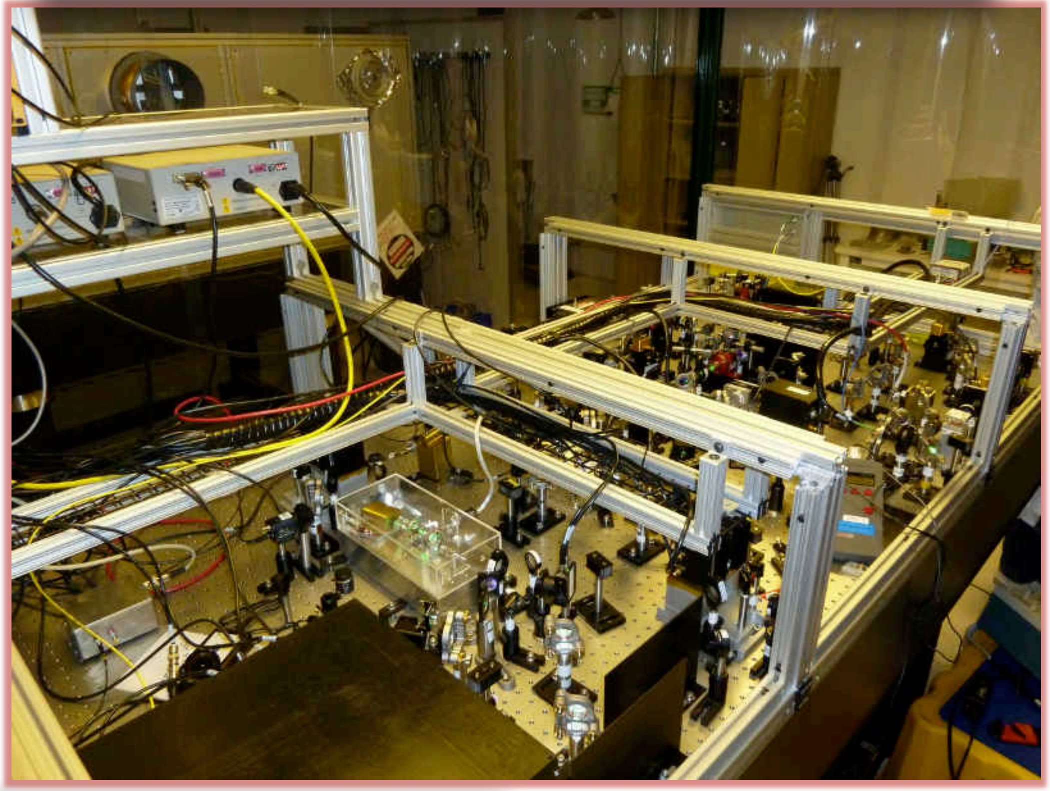
LIGO, Nature Physics (2011)

Squeezed Light in Action: LIGO 4km



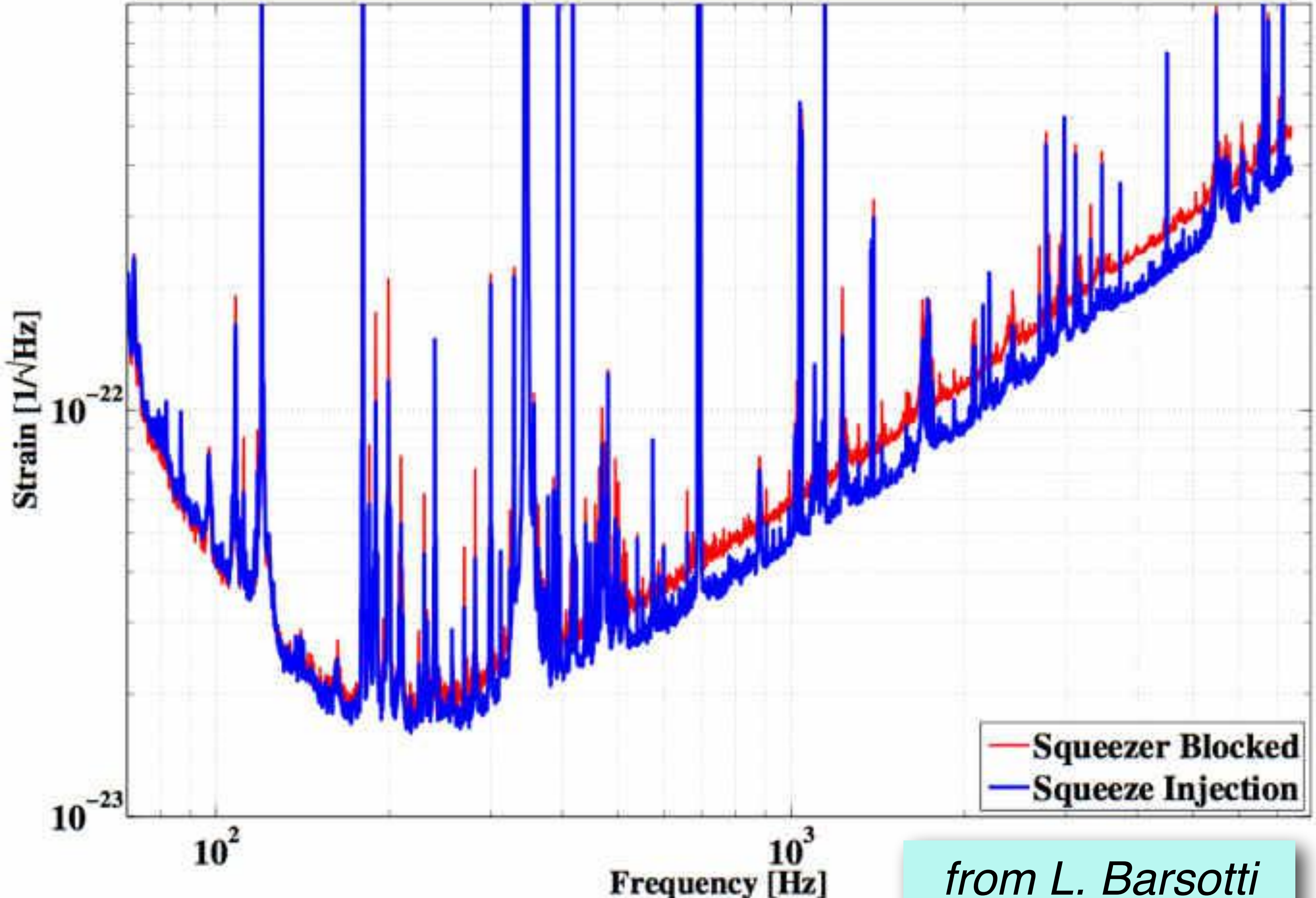
Optical Parametric Oscillator (from ANU)

Squeezed Light Injection Table



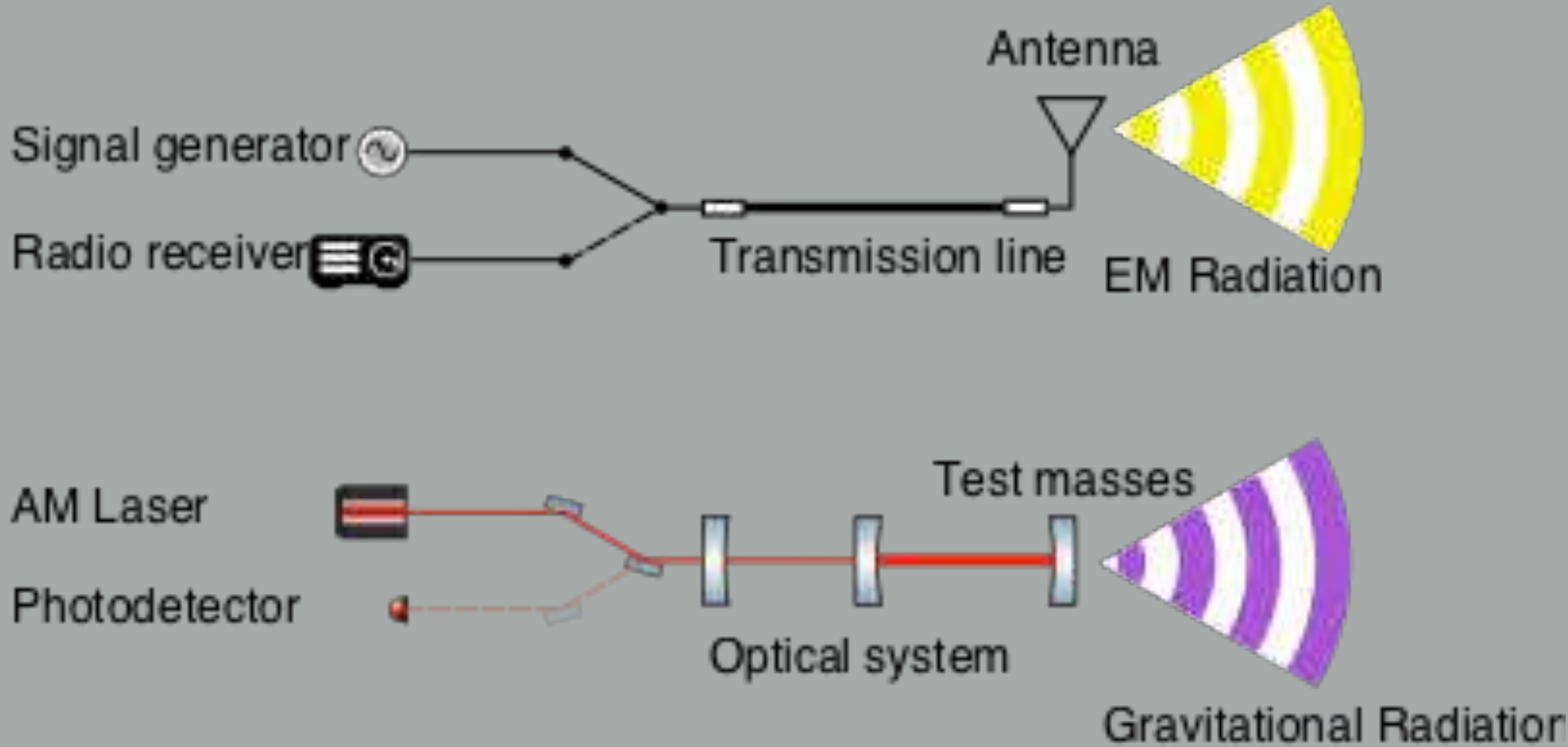
Installed at LIGO Hanford throughout 2011

Squeezed Light in Action: LIGO 4km

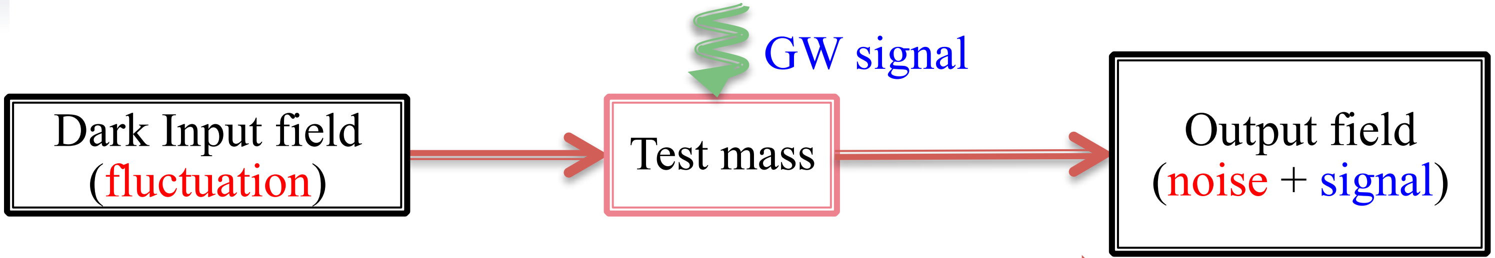


from L. Barsotti

GOOD RECEIVER = GOOD RADIATOR

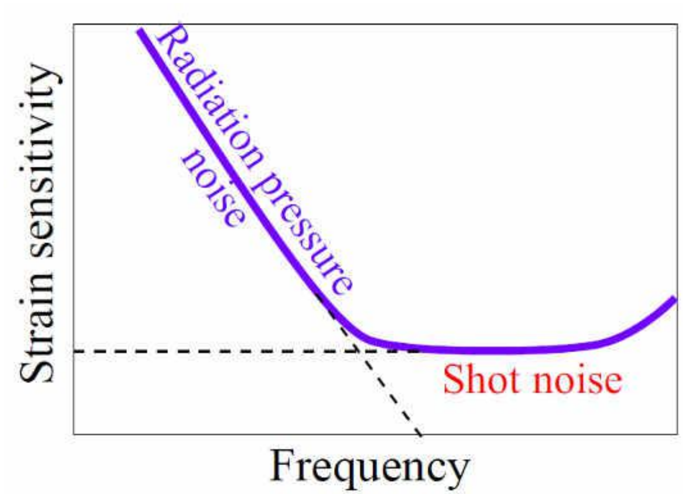


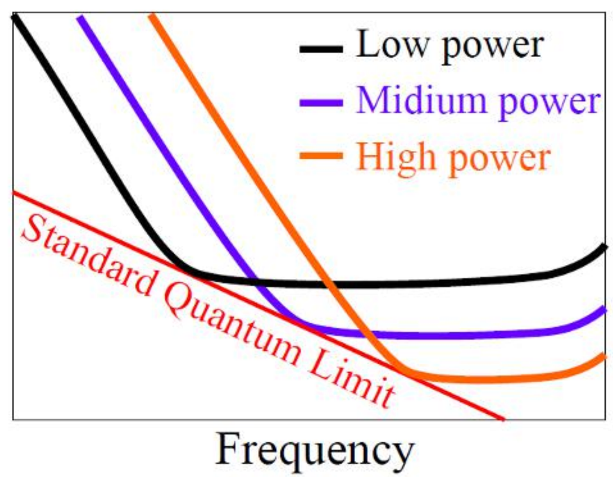
Quantum Noise and SQL



Quantum fluctuation in phase quadrature ⇨ **Shot noise**

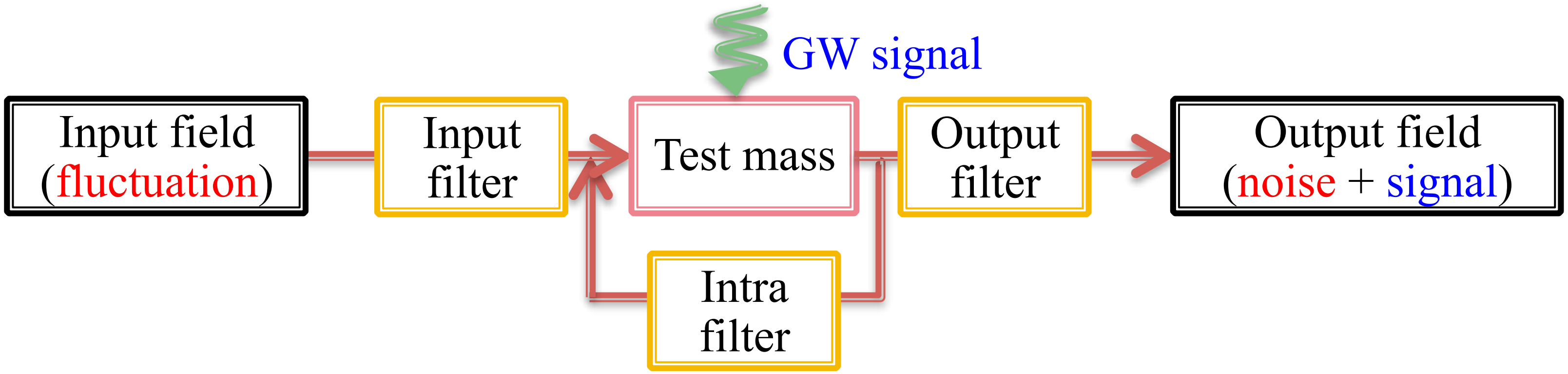
In amplitude quadrature ⇨ **Radiation pressure noise**





Standard Quantum Limit: $S_{hh}^{\rm SQL}(\Omega) = \dfrac{8\hbar}{M\Omega^2 L_{\rm arm}^2}$ $\quad S_{xx}(\Omega) = \dfrac{8\,\hbar}{M\Omega^2}$

General filtering schemes



Each filter can be cascades of several filter cavities.

- - - - - - ***Thousands* of possible combinations**.

Questions:

No 1: How do we implement filters in a **systematic** way?

No 2: What do we care about? Astrophysics? SNR?

For known GW waveform:

$$\mathrm{SNR} = \int \frac{|h_{\mathrm{GW}}(\Omega)|^2}{S_{hh}(\Omega)} \mathrm{d}\Omega$$

Strain noise PSD $S_{hh}(\Omega)$

No 3: What is the **fundamental quantum limit** on SNR? ***Not* the SQL**

Energetic (Fundamental) Quantum Limit

Energetic quantum limit by Braginsky *et al.* [1]:

$$\mathrm{SNR} \le \frac{1}{\hbar^2}\langle \hat{\mathcal{A}}_{\rm int}^2\rangle = \frac{1}{\hbar^2}\langle [\int \mathrm{d}t \hat{H}_{\rm int}(t)]^2\rangle$$

In terms of frequency-domain spectrum:

$$\mathrm{SNR} = \int \frac{\mathrm{d}\Omega}{2\pi}\frac{|h(\Omega)|^2}{S_{hh}(\Omega)} \le \frac{L_{\rm arm}^2}{\hbar^2 c^2}\int \frac{\mathrm{d}\Omega}{2\pi}|h(\Omega)|^2 S_{PP}(\Omega)$$

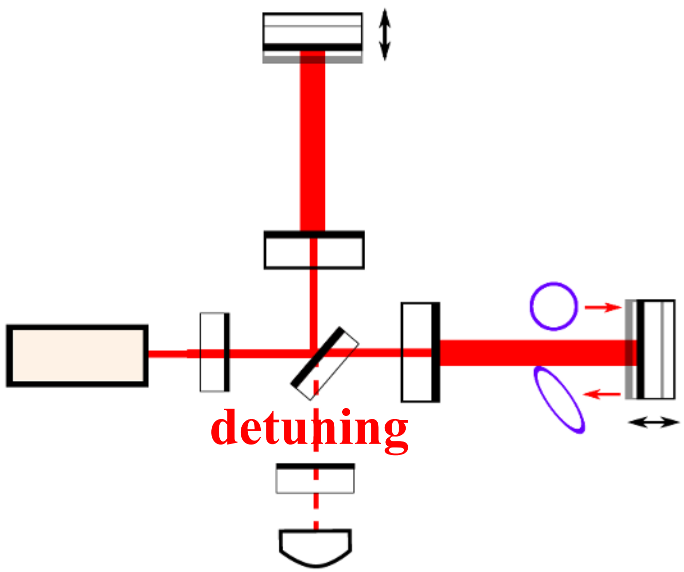
$$\Rightarrow \int \frac{\mathrm{d}\Omega}{2\pi}\frac{1}{S_{hh}(\Omega)} \le \frac{L_{\rm arm}^2}{\hbar^2 c^2}\Delta P_{\rm arm}^2$$

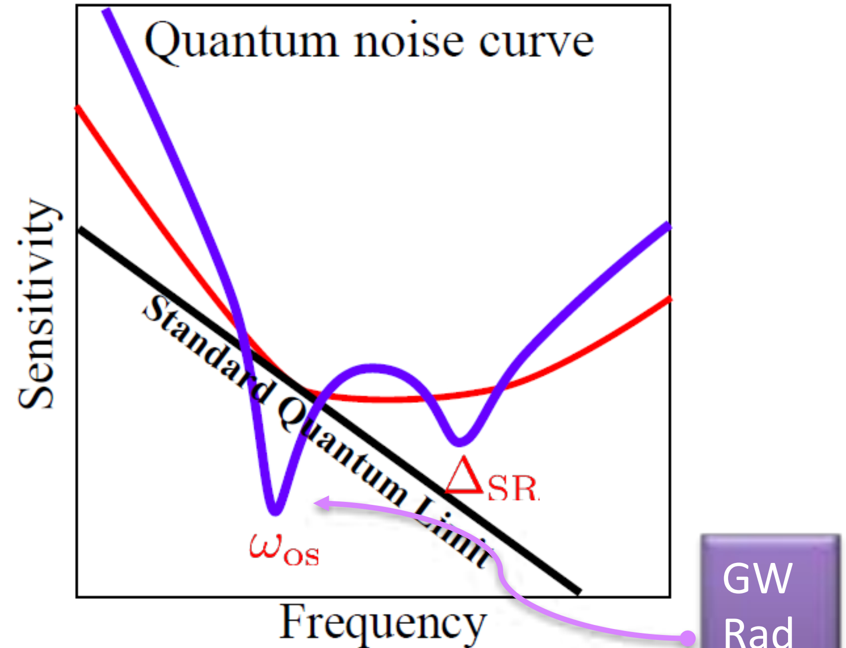
A general bound

[1] V. Braginsky, M. Gorodetsky, F. Khalili, and K. Thorne, *Energetic quantum limit in large-scale interferometers* (1999).

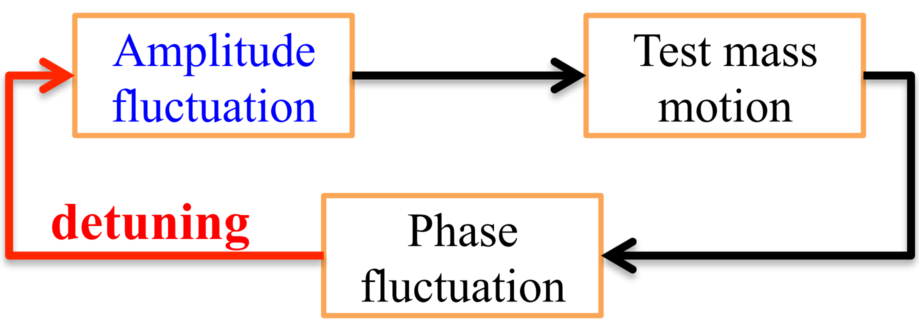
A new perspective on optical springs

Detuned signal recycling:





Coherent optical feedback:



$$G_{\mathrm{CL}}(\Omega) = \frac{1}{1 - G_{\mathrm{OL}}(\Omega)}$$

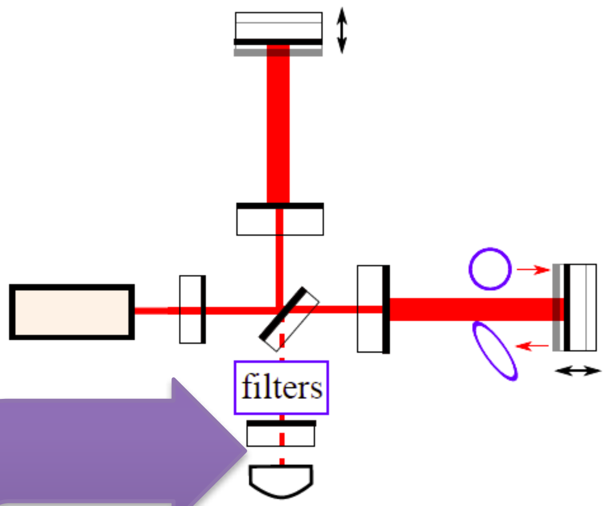
$$|G_{\mathrm{OL}}(\omega_{\mathrm{os}})| = 1$$

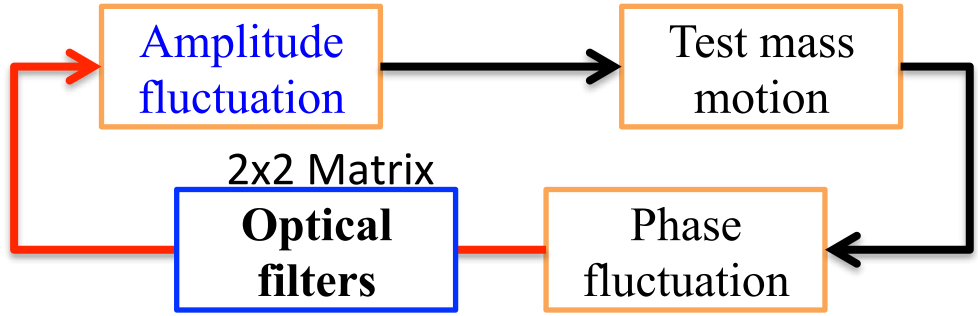
$S_{PP}(\omega_{\mathrm{os}})$ **is very large.**

Upper & lower sidebands almost balance around ω_{os} but not at Δ_{SR}

Coherent optical feedback

Include ponderomotive squeezing as a part of the design:





$$G_{\mathrm{CL}}(\Omega) = \frac{1}{1 - |G_{\mathrm{OL}}(\Omega)| e^{i\phi(\Omega)}}$$

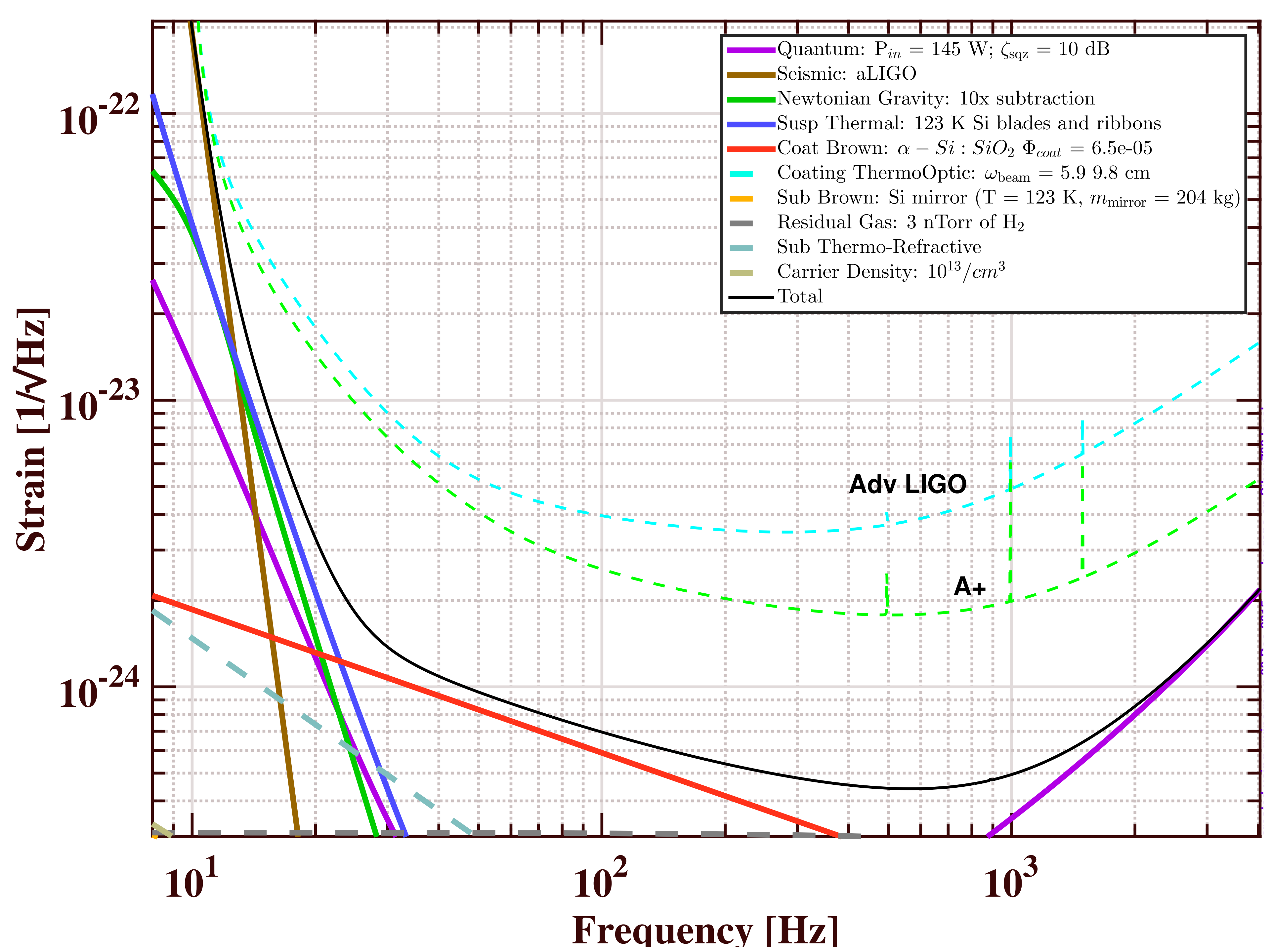
$$|G_{\mathrm{OL}}(\Omega)| \approx 1$$

$$\phi(\Omega) \approx 0$$

Technical issues:

- **Shaping of gain and phase of optical feedback.**
- **Upper and lower sidebands balancing (equal fluctuation/response).**
- **Freq dep. readout angle if response is highly frequency dependent.**
- **Optical loss as usual**

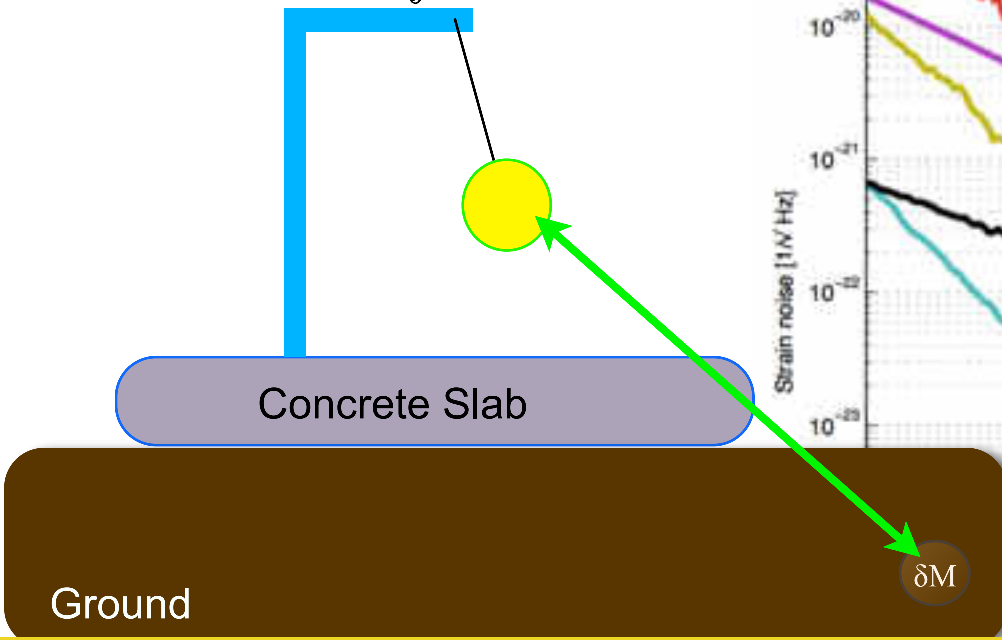
5%

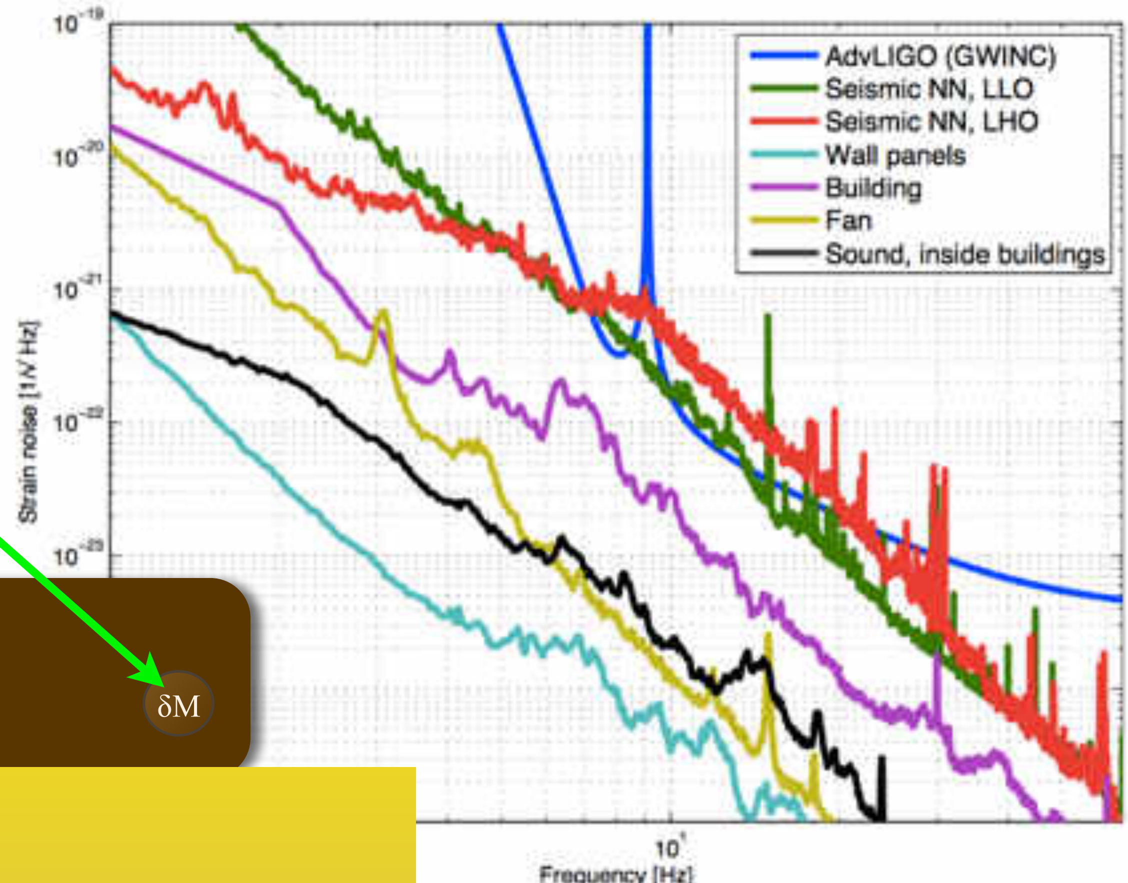

Quantum: P_{in} = 145 W; ζ_{sqz} = 10 dB
Seismic: aLIGO
Newtonian Gravity: 10x subtraction
Susp Thermal: 123 K Si blades and ribbons
Coat Brown: $\alpha - Si : SiO_2$ Φ_{coat} = 6.5e-05
Coating ThermoOptic: ω_{beam} = 5.9 9.8 cm
Sub Brown: Si mirror (T = 123 K, m_{mirror} = 204 kg)
Residual Gas: 3 nTorr of H_2
Sub Thermo-Refractive
Carrier Density: $10^{13}/cm^3$
Total
Adv LIGO
A+
10^{-22}
10^{-23}
10^{-24}
Strain [1/√Hz]
10^1
10^2
10^3
Frequency [Hz]

Gravitational Noise

(Newtonian) Gravity Noise

$$x(f) = G\frac{\delta M(f)}{r^2 f^2}$$

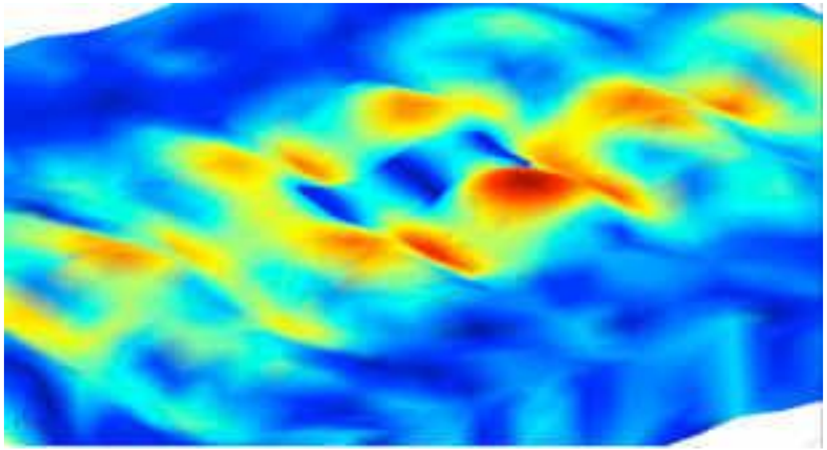




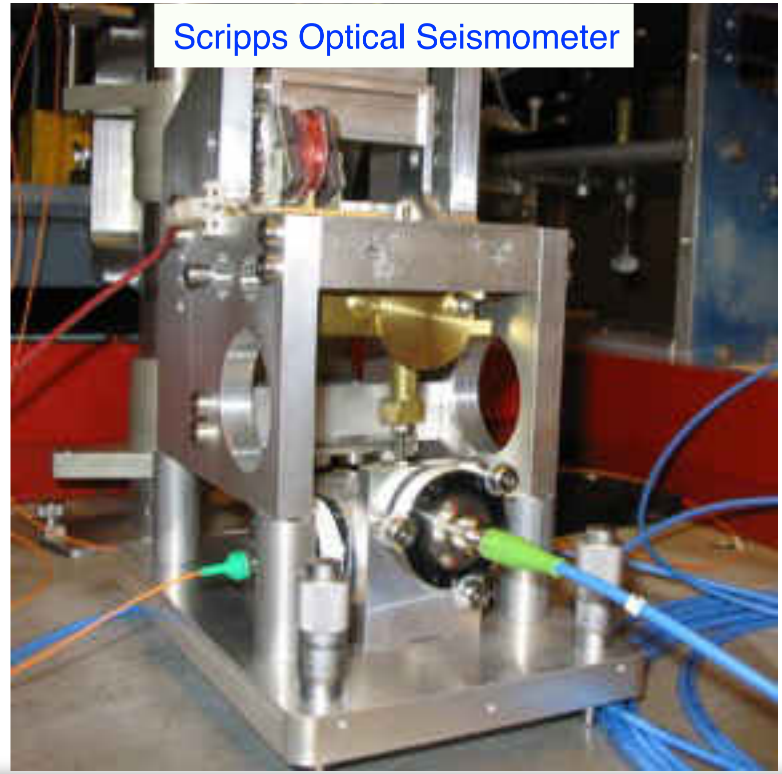
- **Noise Sources**
  - **Surface Waves**
  - **Air Pressure Fluctuations**
  - **Subsurface density Fluctuations**

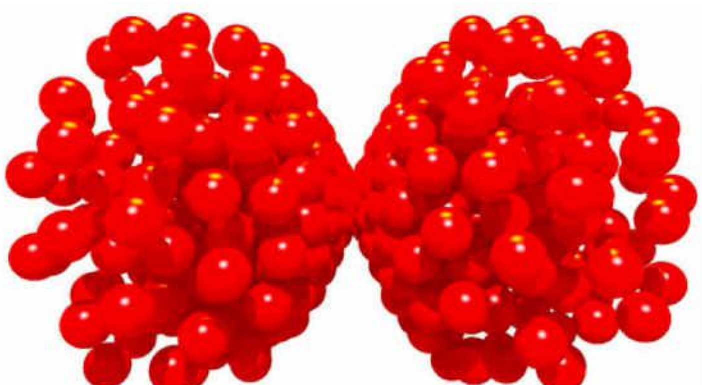
Hughes and Thorne, PRD (1998)

Driggers, Harms, and **RA**, in prep. (2012)



FEA of Ground, Buildings





3D array of seismometers

Driggers, Evans, Pepper, and **RA**, RSI (2012)

Derosa, Driggers, Atkinson, Miao, Frolov, Landry, and **RA**, RSI (2012)

- **Adaptive Noise Cancellation**
  - Accelerometer network measures ground motion
  - Adaptive algorithm estimates Gravity noise
  - Proof of principle demonstrated on Caltech 40m
  - Used in LIGO S6 to save months of duty cycle
- **Next Phase**
  - **Extended site surveys** of seismic / acoustics
  - Detailed FEM of structures / fields
  - Use modern Machine Learning techniques